**IMAP^2 WORKSHOP YEAR 3 SUMMER 2009**

**EQUATION-SOLVING & NUMBER PATTERNS INVESTIGATIONS**

**PART 2: FORMULAS AND EQUATION SOLVING**

**FIBONACCI (?FIBONACC-BEE?) NUMBERS:**



In the honey bee and the cow problems, the sequence of numbers generated comes from the well-known Fibonacci sequence:

1, 1, 2, 3, 5, 8, 13, 21, …

Let ***Fn*** denote the *n*th number in this sequence. So

***F1*** = 1

***F2*** = 1

***F3*** = 2

***F4*** = 3

***F5*** = 5

…

In general, ***Fn+2 = Fn+1 + Fn***and this ‘recursion’ rule together with specifying the first two numbers *F1* and *F2*uniquely determines the entire sequence.

Fibonacci Sequence

***F1*** = 1

***F2*** = 1

***Fn+2 = Fn+1 + Fn***

**Part** I.

**1.1.** Consider the 'family history' of a male bee. He must have just one parent, a female, his mother. Starting with the initial male bee as ***F1*** = 1, the preceding generations will have this many bees (adding together male and female bees in that generation):

***F1*** = 1

***F2*** = 1

***F3*** = 2

***F4*** = 3

***F5*** = 5

***F6*** = 8

***F7*** = 13

…

This certainly looks like the Fibonacci sequence! What ideas might you need to try to make a convincing argument (a.k.a. proof?) that we do indeed get the Fibonacci sequence in this setting?

If you knew exactly how many male and female bees there were in a given generation, could you determine the numbers of males and females in the previous generation?

**1.2.** Is there a ‘formula’ for the Fibonacci sequence? Such a formula might allow to compute terms in the Fibonacci sequence without having to compute all the previous terms. For example, is there a ‘simple’ formula for ***F100*** which does not require knowing or computing all the previous 99 Fibonacci numbers ***F1,***  ***F2,***  ***F3,***  ..., ***F99*** ?

The Fibonacci sequence can be considered an example of a *discrete dynamical system*, a topic in the subject area of mathematics called discrete mathematics. The topic of discrete mathematics is highlighted in the NCTM PSSM and the Iowa Core Curriculum

as an important (and generally underserved) area of mathematics in the K-12 mathematics curriculum.

One might define a *discrete dynamical system* as a set of equations involving one or several real-valued functions, each with domain the natural numbers. These functions have the property that their values at a given natural number *n* are determined by their values at some or all of the previous inputs *n-1, n-2, n-3, ….4, 3 , 2, 1*. In many cases the system includes some ‘initial conditions’ which specify starting values (*n = 1*) for the functions involved.

The word ‘discrete’ refers to the idea that the values of the functions involved depend on the discrete set of real numbers 1, 2, 3, …. The word ‘dynamical’ refers to the idea that the values of the functions involved are changing (hence “dynamic”) as the inputs change.

General Discrete Dynamical System (one function)

***Fn = Function of { Fn-1 , Fn-2 , Fn-3 ,… F2 , F1 }***

Example Discrete Dynamical System (Fibonacci)

Let *A* be a real number

***Fn+2 = Fn+1 + Fn***

***F1 = 1***

***F2 = 1***

One of the simplest examples is the geometric sequence. Let *A* and *r* be two real numbers

The geometric sequence/discrete dynamical system

Let *A* and *r* be two real numbers

***F1*** = *A*

***Fn = r\* Fn-1***

The sequence of numbers generated by this system is

{ *A, A\*r, A\*r2, A\*r3, …}*

For this d.d.s. there is a formula for***Fn*** allows one to compute terms in the sequence without having to compute all the previous terms. This formula can be obtained in many different ways (observation, guessing, trial and error, etc.)

The geometric sequence/discrete dynamical system

***F1*** = *A*

***Fn = r\* Fn-1***

***Closed form expression for Fn***

***Fn = A\*rn***

The geometric sequence or the power function closed form solution might be called a ‘basic building block’ for discrete dynamical systems. This is very similar to the way that the exponential function *ex* is a ‘basic building block’ for differential equations/ systems.

The exponential function/differential equation

Let A and to be a real numbers.

 A real-valued function F of a real number variable t is said to satisfy the exponential function/differential equation/ system of it satisfies the two equations:

***F[to*** ***] = A***

***F’[t*** ***] = r\* F[t*** ***]***

***for all real numbers t.***

***A closed form expression for F[t*** ***]is***

 ***F[t*** ***] = A \* e*** *r \***( t* ***-*** *to)*

It is possible to derive a ‘formula’ for the Fibonacci sequence by using the ‘basic building block’ d.d.s. solution as a starting off guess as a function which does generate the Fibonacci sequence. This initial guess doesn’t work (in most cases) but writing out what you get may suggest how to modify this guess to get a Fibonacci sequence.

Try this out and see what happens! Specifically, suppose you guess that an expression of

***Fn = A\*rn***

‘obeys’ the Fibonacci sequence rules. What does that tell you about the *A, r* and *n*, if anything? What do you do next?