**IMAP^2 WORKSHOP YEAR 3 SUMMER 2009**

**EQUATION-SOLVING & NUMBER PATTERN INVESTIGATIONS**

**PART 1: BEES AND COWS**

In almost all mathematics problems, including problems arising from applications of mathematics to modeling real-world phenomena, an important component of the problem and its solution is solving equations (and/or inequalities). In these investigations we examine questions about existence and uniqueness of solutions to equations in some familiar settings and some which might not be as familiar.

**I. HONEY BEES:**

There are three types of honey bees: queens, female worker bees and male drone bees. Female honey bees have two (2) parents, a male and a female whereas male bees have just one parent, a female. The queen lays an egg and if that egg is fertilized, it will grow into a female. If the egg is unfertilized it will grow into a male.

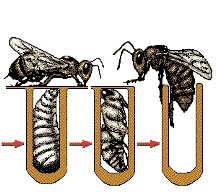
There is a lot of information about honey bees on a variety of web sites for example from the Everything About website at url: http://www.everythingabout.net/

Search under Everything About Bees for Honey Bees.

http://www.everythingabout.net/articles/biology/animals/arthropods/insects/bees/honey\_bee/more\_honey\_bee.shtml

We include a quote below about honey bees from this web site.

### "More about Honey Bee

[](http://www.everythingabout.net/articles/biology/animals/arthropods/insects/bees/honey_bee/honey_bee_from_egg_to_adult.shtml" \t "_blank)

[click to enlarge](http://www.everythingabout.net/articles/biology/animals/arthropods/insects/bees/honey_bee/honey_bee_from_egg_to_adult.shtml" \t "_blank)

Microsoft Illustration

Reproduction and Development   
The queen controls the sex of her offspring. When an egg passes from her ovary to her oviduct, the queen determines whether the egg is fertilized with sperm from the spermatheca (an organ in the queen which stores sperm from male drone bees). A fertilized egg develops into a female honey bee, either worker or queen, and an unfertilized egg becomes a male honey bee, or drone.

The queen lays the eggs that will develop into more queens in specially constructed downward-pointing, peanut-shaped cells, in which the egg adheres to the ceiling. These cells are filled with royal jelly to keep the larvae from falling and to feed them.

Worker bees are raised in the multi-purpose, horizontally arranged cells of the comb. Future workers receive royal jelly only during the first two days, compared to future queens, who are fed royal jelly throughout their larval life. This difference accounts for the great variation in anatomy and function between adult workers and queens. On average, the development of the queen from egg to adult requires 16 days; that of the worker, 21 days; and that of the drone, 24 days."

**I.1. Investigation 1-Male bee family history**

**I.1.1.**Investigate the 'family history' of a male bee. He must have just one parent, a female, his mother. The mother must have a mother and a father, the original bee's grandmother and grandfather. How many great parents, great-great grandparents, etc. must the starting male bee have?

If we just count the total number of bees (males and females) in each generation by itself, what sequence of numbers does that generate?

Are there any assumptions we have to make to answer these questions, assumptions we have not already made explicit?

**I.1.2.** Is there a good graphical, tabular, numerical or algebra-formula method for keeping track of how many ancestors in a given generation the male bee has? Is there a pattern to this total number of ancestors?

For example, if we go back ten generations in the starting male bee's family history, how many bees will there be in that tenth generation? What if we go back 50 generations?

**I.1.3.** Is there a good graphical, tabular, numerical or algebra-formula method for keeping track of how many male ancestors in a given generation the male bee has? Is there a good graphical, tabular, numerical or algebra-formula method for keeping track of how many female ancestors the male bee has?

**I.2. Investigation 2-Female bee family history**

**I.2.1.** Investigate the 'family history' of a female bee. She must have just two parents, a female, her mother and a male, her father. The mother must have a mother and a father, the original bee's grandmother and grandfather. And the starting female bee must also have a male father bee, who in turn would have just a mother. So the starting female bee would have three grandparents-two grandmothers and one grandfather. How many great parents, great-great grandparents, etc. must the starting female bee have?

If we just count the total number of bees (males and females) in each generation by itself, what sequence of numbers does that generate?

Are there any assumptions we have to make to answer these questions, assumptions we have not already made explicit?

**I.2.2.** Is there a good graphical, tabular, numerical or algebra-formula method for keeping track of how many ancestors in a given generation the female bee has?? Is there a pattern to this total number of ancestors?

For example, if we go back ten generations in the starting female bee's family history, how many bees will there be in that generation? What if we go back 50 generations?

**I.2.3.** Is there a good graphical, tabular, numerical or algebra-formula method for keeping track of how many male ancestors the female bee has? Is there a good graphical, tabular, numerical or algebra-formula method for keeping track of how many female ancestors the male bee has?

**I.3. Investigation 3-Honey bee family history in your classrooms**

**I.3.1.** Are there parts of the honey bee investigations which would be useful in developing improved math and/or problem-solving skills for your students?

**I.3.2.** What are some tasks you might pose with your students about honey bees' reproduction cycles. What parts of those tasks do you think students would find most challenging?

**I. 3.3.** What math, problem-solving or other questions do you think students would ask about the honey bee investigations? What would your responses to those questions be (bee)?

**II.COWS:**

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Say a female calf, Elsie, is born say on January 1 of 2008.

Two years after Elsie's birth, on January 1 of 2010, she gives birth to her own first calf, which like Elsie is also a female. Then on January 1 of 2011, 2012, 2013, and every year after that, Elsie always produces one new female calf.

Suppose every calf born to Elsie produces her first female calf at the end of two years, on January 1, and after that produces another single female calf every year (again on January 1).

**II.1 Investigation 1-Elsie's descendents**

**II.1.1.** How many total cows and calves are there after 8 years, 12 years and 15 years starting from when Elsie was born (say on January 1)? Assume none of the calves born during this time die. Include the starting cow Elsie in this total number of cows and calves.

**II.1.2.** Is there a good graphical, tabular, numerical or algebra-formula method for keeping track of how many descendents Elsie has? Is there a pattern to this total number of descendents?

**II.2. Investigation 2-Elsie descendents in your classrooms**

**II.2.1.** Are there parts of the Elsie investigations which would be useful in developing improved math and/or problem-solving skills for your students?

**II.2.2.** What are some tasks you might pose with your students about Elsie's descendents? What parts of those tasks do you think students would find most challenging?

**II. 2.3.** What math, problem-solving or other questions do you think students would ask about the Elsie investigations? What would your responses to those questions be (bee)?

**III. BEES, COWS AND CONNECTIONS**

**III.1. Investigation 3 Connections between Elsie’s descendents and the honey bees**

**III. 1.1.** What parts of the problems involving Elsie's descendents are related in some way to the problems investigating the honey bee ancestors? How might you explain the mathematical connections between the Elsie problems and the honey bee problems?

**III. 1.2.** What connections do you think students in your classes might make between the problems involving Elsie's descendents and the problems investigating the honey bee ancestors? What questions do you think they might ask about trying to make such connections, and how would you answer those questions?