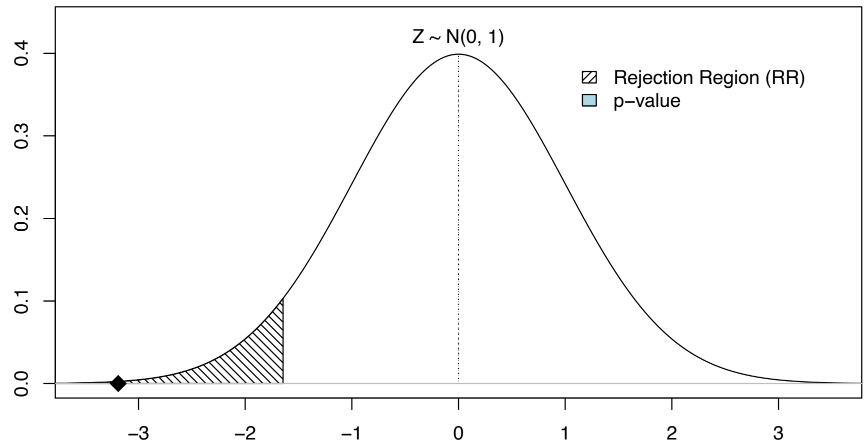

Figure 9.4: Comparison of 2007 and 1978 Pennies: Test statistic z^* is denoted by ◆, the rejection region and p−value are also shown. Note that the total area in the rejection region is equal to $\alpha = 0.05$.

(2) Critical Value: This is a one-sided test, so

$$z_\alpha = z_{0.05} = 1.645$$

(3) Decision: See Figure 9.4. Reject H_0. Evidence that $p_1 < p_2$. The proportion of heads with the 2007 penny is significantly lower than the 1978 penny.

(d) Find the p−value for the test in (c).

$$p - \text{value} = P(Z < z^*) = P(Z < -3.19) = 0.0007$$

Since the p−value is less than α, reject H_0. The small p−value indicates that we have strong evidence that the 2007 penny yields a significantly lower proportion of heads than the 1978 penny.

9.3 Exercises

♥ = answers are provided beginning on page 229.

9.1 ♥ Let p denote the population proportion of (budget) LCD panels that have defective pixels. A researcher randomly selected 100 LCD panels and determined that 8 have defective pixels.

(a) Suppose we wish to test $H_0 : p = 0.12$ versus $H_a : p \neq 0.12$ at the $\alpha = 0.05$ significance level using the score test. Approximate the p−value for this test.

(b) In reference to 9.1(a), does the proportion of defective panels significantly differ from 0.12? Why?

(c) Find an approximate 95% Wald confidence interval for p.

(d) Based upon your answer in 9.1(c), does the proportion of defective panels significantly differ from 0.12? Why?

(e) What is the estimated standard error of $\hat{p}$, $\widehat{se}(\hat{p})$?

(f) What is the estimated margin of error (at 95% confidence)?

(g) How many panels would need to be selected for the margin of error (at 95% confidence) to equal 0.03?

9.2 ♥ Suppose the population proportion of cellular customers that use more than 5GB of data per month is p. A random sample of 32 customers had 10 that exceeded 5GB of data.

(a) Find an approximate 95% Agresti-Coull confidence interval for p.

(b) Based upon your answer in 9.2(a), does p significantly differ from 0.10? Why?

(c) Based upon your answer in 9.2(a), does p significantly differ from 0.40? Why?

(d) Suppose we wish to test $H_0 : p = 0.20$ versus $H_a : p \neq 0.20$ at the $\alpha = 0.05$ significance level. Based upon your answer in 9.2(a), will the $p-$value for the test be less than or greater than α? Why?

9.3 Suppose the population proportion of computer keyboards that fail is p. To infer about p, a manufacturer randomly selected 60 keyboards and determined that 6 had failed.

(a) Find an approximate 95% Wald confidence interval for p.

(b) Find an approximate 95% Agresti-Coull confidence interval for p.

(c) Based upon your answer in 9.3(b), does p significantly differ from 0.15? Why?

(d) Based upon your answer in 9.3(b), does p significantly differ from 0.50? Why?

(e) Suppose we wish to test $H_0 : p = 0.50$ versus $H_a : p \neq 0.50$ at the $\alpha = 0.05$ significance level. Based upon your answer in 9.3(b), will the $p-$value for the test be less than or greater than α? Why?

9.4 A retailer is seeking to study the satisfaction of its customers. The population proportion of satisfied customers is p. A random sample of 100 customers yielded 90 that were satisfied. We would like to determine if more than 80% of the customers are satisfied.

(a) Test $H_0 : p = 0.8$ versus $H_a : p > 0.8$ at the $\alpha = 0.05$ significance level using the score test. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Find the $p-$value for the test in 9.4(a).

(c) Based upon your answer in 9.4(b), is the proportion of satisfied customers significantly higher than 0.80? Why?

9.5 ♥ When LED's are used in automobile tail lights, the population proportion that fail prematurely is equal to p. Suppose 90 out of 500 randomly selected LED's failed prematurely in this particular application.

(a) Suppose a manufacturer wants to test $H_0 : p = 0.2$ versus $H_a : p < 0.2$ at the $\alpha = 0.01$ significance level using the score test. Determine the $p-$value for this test.

(b) Suppose a manufacturer wants to test $H_0 : p = 0.15$ versus $H_a : p < 0.15$ at the $\alpha = 0.01$ significance level using the score test. Determine the $p-$value for this test.

9.6 An art store receives their glass (for framing pictures) from an overseas supplier. The supplier sends sheets of glass in quantities of 1,000 pieces. Because the quality of the glass is so important, the art store must discard any sheets of glass that are scratched. Because of this, the art store won't accept any shipments where more than 20% of the sheets are scratched. Because inspecting all 1,000 sheets is impractical, the store randomly chooses 50 sheets of glass for inspection. If 15 of the 50 sheets are scratched, should the art store reject the shipment of glass? *In your answer, let p denote the population proportion of scratched sheets in a shipment (of 1,000 sheets).*

(a) (*4 pts*) To answer this question, we must perform a one-sided hypothesis test. How should H_0 and H_a be specified?

(b) (*6 pts*) Perform the test in 9.6(a) at the $\alpha = 0.10$ significance level using the score test. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(c) (*2 pts*) Based upon your answer in 9.6(b), should the store reject the shipment of glass? Why?

(d) (*4 pts*) Find the $p-$value for the test in 9.6(a).

(e) (*2 pts*) Based upon your answer in 9.6(d), is there evidence that the proportion of scratched sheets in the shipment is higher than 0.20? Why?

(f) (*2 pts*) If the significance level $\alpha = 0.01$ (instead of 0.10), should the shipment be rejected? Why?

9.7 ♥ A retailer is seeking to study the satisfaction of its customers. The population proportion of satisfied *male* customers is p_1, while the population proportion of satisfied *female* customers is p_2. A random sample male and female customers yielded

$$\begin{aligned} \text{Male:} \quad & n_1 = 50, \ \text{number of satisfied customers} = 34 \\ \text{Female:} \quad & n_2 = 70, \ \text{number of satisfied customers} = 61 \end{aligned}$$

(a) (*6 pts*) Test $H_0 : p_1 = p_2$ versus $H_a : p_1 < p_2$ at the $\alpha = 0.01$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Based upon your answer in 9.7(a), is a significantly higher proportion of females satisfied than males? Why?

(c) Find the $p-$value for the test in 9.7(a).

9.8 The proportion of male Americans that invest in the stock market is p_1, while the proportion of females that invest is p_2. A random sample of 40 males had 18 investors, while a random sample of 50 females had 12 investors.

(a) Find a 95% confidence interval for $p_1 - p_2$.

(b) Based on your answer in 9.8(a), is there a significant difference in the proportion of male and female investors? Why?

(c) Suppose we wish to test $H_0 : p_1 = p_2$ versus $H_a : p_1 \neq p_2$ at the $\alpha = 0.05$ significance level. Find the $p-$value for this test.

(d) Based on your answer in 9.8(c), is there a significant difference in the proportion of male and female investors? Why?

(e) Suppose we wish to test $H_0 : p_1 = p_2 + 0.1$ versus $H_a : p_1 \neq p_2 + 0.1$ at the $\alpha = 0.05$ significance level. Find the $p-$value for this test.

9.9 After smoking marijuana, 7 out of 100 subjects failed a driving test on the Iowa Driving Simulator. Only 7 out of 140 subjects *not* under the influence of marijuana failed the test. Let p_1 denote the population proportion of marijuana users that fail the test, and let p_2 denote the population proportion of drivers *not* under the influence of marijuana that fail the driving test.

(a) Find an 80% confidence interval for $p_1 - p_2$. Is there a significant difference between the groups? Why?

(b) Suppose we wish to test $H_0 : p_1 = p_2$ versus $H_a : p_1 \neq p_2$. Find the $p-$value for this test. Is there a significant difference in the proportion of marijuana users and non-marijuana users fail the test? Why?

9.10 A high-quality diamond cutter can receive raw diamonds from two mines: one is in Russia and the other is in Africa. The Russian mine charges slightly more for its diamonds. However, if the Russian mine has a higher proportion of "colorless" diamonds, the diamond cutter will choose to buy his diamonds from them. Let p_1 denote the proportion of colorless diamonds from the Russian mine, and let p_2 denote the proportion of colorless diamonds from the African mine. A random sample of 50 diamonds from each mine found the following:

$$\begin{aligned} \text{Russia:} \quad & n_1 = 50, \text{ number of colorless} = 10 \\ \text{Africa:} \quad & n_2 = 50, \text{ number of colorless} = 6 \end{aligned}$$

(a) Test $H_0 : p_1 = p_2$ versus $H_a : p_1 > p_2$ at the $\alpha = 0.05$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Find the $p-$value for the test in 9.10(a).

(c) Based upon your analysis, is it worthwhile for the diamond cutter to pay extra for diamonds from the Russian mine? Why?

9.11 ♥ Suppose a 95% confidence interval for $p_1 - p_2$ is $(0.26, 0.40)$. A researcher wants to test $H_0 : p_1 - p_2 = 0.25$ versus $H_a : p_1 - p_2 \neq 0.25$ at the $\alpha = 0.05$ significance level. Find the $p-$value for this test.

9.12 Suppose a 95% confidence interval for $p_1 - p_2$ is $(0.43, 0.51)$. A researcher wants to test $H_0 : p_1 - p_2 = 0.5$ versus $H_a : p_1 - p_2 \neq 0.5$ at the $\alpha = 0.05$ significance level. What is the $p-$value for this test?

9.13 The proportion of decaffeinated cups of coffee sold at a coffee shop is p. To infer about p, a worker randomly selected 100 customers that ordered coffee. Out of the 100 customers, 20 ordered decaffeinated. The worker quoted a Wald CI for p to be $(0.1152, 0.2848)$. What percent confidence interval is this?

9.14 Based upon a random sample of $n = 100$ individuals, a researcher tested $H_0 : p = 0.5$ versus $H_a : p > 0.5$ (using the score test) and determined that the $p-$value for the test was 0.0749. Find a 95% Wald confidence interval for p. *Hint: You need to first find the sample proportion $\hat{p}$ from the given information.*

9.15 Suppose a researcher found a 95% Wald confidence interval for p to be $(0.3775, 0.6225)$. She wishes to test $H_0 : p = 0.6$ versus $H_a : p \neq 0.6$ using the score test. Find the p–value for this test. *Hint: First find $\hat{p}$, then find n, then find the test statistic and p–value.*