(g) How large would the sample size n have to be for the power of the test against the alternative $\mu = 44$ to be (at least) 95%?

Using a similar calculation as above, it can be shown that $n = 6.956^2 = 48.389$ *which we round up to* $n = 49$. *Verify this.*

7.5 Exercises

♥ = answers are provided beginning on page 229.

7.1 ♥ The longevity of truck tires (in months) has a normal distribution with mean μ months and standard deviation $\sigma = 8.0$ months. Suppose $n = 16$ tires are randomly selected and the sample mean longevity $\bar{x} = 42.5$ months.

(a) Test $H_0 : \mu = 40$ versus $H_a : \mu \neq 40$ at the $\alpha = 0.10$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Based upon your answer in 7.1(a), does the mean longevity μ significantly differ from 40? Why?

(c) Find a 90% CI for the mean longevity μ.

(d) Based upon your answer in 7.1(c), will the p−value for the test in 7.1(a) be less than α or greater than α? Why?

(e) Find the p−value for the test in 7.1(a).

(f) Based upon your answer in 7.1(c), does the mean longevity μ significantly differ from 55? Why?

(g) How many tires would be needed for $se(\bar{x})$ to equal 1.0?

(h) Even though the sample size $n < 30$, we were able to perform the above analysis. Why?

7.2 ♥ In reference to question 7.1, suppose we wish to perform the one-sided test $H_0 : \mu = 48$ versus $H_a : \mu < 48$ at the $\alpha = 0.01$ significance level.

(a) Perform this test. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Find the p−value for the test.

(c) Based upon your answer in 7.2(b), is the mean longevity significantly less than 48 months? Why?

7.3 The diastolic blood pressure, X, of smokers follows a normal distribution with mean μ and standard deviation $\sigma = 15$, i.e. $X \sim N(\mu, \sigma = 15)$. The diastolic blood pressure of 3 randomly selected smokers was:

`125 140 125`

(a) Find a 95% CI for the population mean diastolic blood pressure μ.

(b) Test $H_0 : \mu = 140$ vs. $H_a : \mu \neq 140$ at the $\alpha = 0.05$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(c) Find the p−value for the test in 7.3(b).

(d) Based upon your answer in 7.3(c), does the population mean diastolic blood pressure μ significantly differ from 140? Why?

(e) Based upon your answer in 7.3(a), does the population mean diastolic blood pressure μ significantly differ from 100? Why?

7.4 In the Iowa Driving Simulator, the number of times the center line is crossed by individuals that are under the influence of alcohol has a distribution that is skewed to the right with mean μ and standard deviation $\sigma = 7$. For the 49 participants that drove after drinking alcohol, the mean number of times the center line was crossed was $\bar{x} = 10$.

(a) Find an approximate 95% confidence interval for μ.

(b) What is the margin of error at (95% confidence)?

(c) Suppose we want to test $H_0 : \mu = 12$ versus $H_a : \mu \neq 12$ at the $\alpha = 0.05$ significance level. Based upon your answer in 7.4(a), is H_0 rejected? Explain.

(d) Test $H_0 : \mu = 12$ versus $H_a : \mu \neq 12$ at the $\alpha = 0.05$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(e) Based upon your answer in 7.4(d), will the p−value for the test be less than α or greater than α? Why?

(f) Find the p−value for the test in 7.4(d).

(g) Based upon your answer in 7.4(f), does the mean number of crossings μ significantly differ from 12? Why?

(h) How many drivers would be needed for the margin of error (at 95% confidence) to equal 0.686?

(i) Could we perform the above analysis if the sample size $n < 30$? Explain.

7.5 In reference to question 7.4, suppose we wish to perform the one-sided test $H_0 : \mu = 9$ versus $H_a : \mu > 9$ at the $\alpha = 0.01$ significance level.

(a) Perform this test. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Based upon your answer in 7.5(a), will the p−value for the test be less than α or greater than α? Why?

(c) Find the p−value for the test in 7.5(a).

(d) Based upon your answer in 7.5(c), is the mean number of crossings μ significantly higher than 9? Why?

7.6 Suppose the weight of bags of M&M's follow a normal distribution with mean μ ounces and standard deviation $\sigma = 0.10$ ounces. A random sample of 4 bags had an average weight $\bar{x} = 16.10$ ounces. Suppose we wish to test $H_0 : \mu = 16$ vs $H_a : \mu > 16$ at the $\alpha = 0.05$ significance level. What is the p−value for this test?

(a) What is the p−value for this test?

(b) Is the mean weight μ significantly more than 16 ounces? Why?

(c) Suppose the significance level $\alpha = 0.01$. Is the mean weight μ significantly more than 16 ounces? Why?

7.7 ♥ The gain of a certain type of JFET transistor follows a normal distribution with mean μ and standard deviation σ. An electrical engineer randomly selected 7 transistors, and computed $\bar{x} = 116.2$ and $s = 7.8$.

(a) Find a 95% confidence interval for μ.

(b) If we were to test $H_0 : \mu = 125$ vs. $H_a : \mu \neq 125$ at the $\alpha = 0.05$ significance level, would you reject H_0? Why?

(c) Test $H_0 : \mu = 125$ vs. $H_a : \mu \neq 125$ at the $\alpha = 0.05$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(d) Find the p–value for the test in 7.7(c).

7.8 The amount of time per day (in hours) office workers spend working on a computer can be modeled by a normal distribution with mean μ and standard deviation σ. A manager wants to infer about the population mean μ, so he randomly selects 5 employees and observes their work habits. The raw data is:

$$6.5, \ 7.1, \ 5.9, \ 6.2, \ 6.3$$

(a) Compute the sample mean $\bar{x}$ and the sample standard deviation s.

(b) Test $H_0 : \mu = 6$ vs. $H_a : \mu \neq 6$ at the $\alpha = 0.01$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(c) Based upon your answer in 7.8(b), does the population mean computer time μ significantly differ from 6 hours? Why?

(d) Find the p–value for the test in 7.8(b).

(e) Find a 99% confidence interval for μ.

(f) Another manager wants to do the one-sided test $H_0 : \mu = 6$ vs. $H_a : \mu > 6$ at the $\alpha = 0.10$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

7.9 Wood et. al (1988) studied the efficacy of diet for losing weight. The study, which lasted one year, involved only men. The weight loss for dieting men follows a normal distribution with mean μ and standard deviation σ. A group of $n = 16$ dieting men lost an average of $\bar{x} = 7.2$ pounds with standard deviation $s = 4.4$ pounds. *This problem will highlight the fact that a one-sided test has more statistical power than a two-sided test.*

(a) Find a 90% confidence interval for μ.

(b) Based upon your answer in 7.9(a), does the population mean weight loss μ significantly differ from 5.5 pounds? Why?

(c) Test $H_0 : \mu = 5.5$ vs. $H_a : \mu \neq 5.5$ at the $\alpha = 0.10$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(d) Approximate the p–value for the test in 7.9(c).

(e) Based upon your answer in 7.9(d), does the population mean weight loss μ significantly differ from 5.5 pounds? Why?

(f) Test $H_0 : \mu = 5.5$ vs. $H_a : \mu > 5.5$ at the $\alpha = 0.10$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(g) Approximate the p–value for the test in 7.9(f).

(h) Based upon your answer in 7.9(g), is the population mean weight loss μ significantly more than 5.5 pounds? Why?

7.10 The amount of energy storage of certain type of capacitor (a small electronic device) has a distribution that is strongly skewed to the left with mean μ pF (pico Farad) and standard deviation $\sigma = 150$ pF. An electrical engineer randomly selected 100 capacitors and determined the CI for μ is $(383, 437)$ pF. What percent confidence interval is this?

7.11 ♥ Suppose cholesterol levels of athletes follow a $N(\mu, \sigma)$ distribution. The average cholesterol level of 31 randomly selected athletes was $\bar{x} = 130.3726$ with standard deviation $s = 12$. Suppose we wish to test $H_0 : \mu = 135$ versus $H_a : \mu < 135$ at the $\alpha = 0.01$ significance level.

(a) Find the p–value for this test.

(b) Is the mean cholesterol level μ significantly less than 135? Why?

(c) Suppose the significance level $\alpha = 0.05$. Is the mean cholesterol level μ significantly less than 135? Why?

7.12 ♥ Suppose we wish to test $H_0 : \mu = 100$ versus $H_a : \mu \neq 100$ at the $\alpha = 0.05$ significance level. Assume the population is normally distributed with mean μ and standard deviation σ (unknown). If the sample size $n = 9$, the sample mean $\bar{x} = 99.4$, and the p–value for the test is 0.02, find the sample standard deviation s.

7.13 The gain of a certain type of MOSFET transistor follows a normal distribution with mean μ and standard deviation $\sigma = 11$. An electrical engineer randomly selected 16 transistors, and determined a CI for μ to be $(71.5, 81.5)$.

(a) What percent confidence interval is this?

(b) How large of a sample size n would be required for the margin of error to equal 2 at 95% confidence? *Round your answer up to the next whole number.*

7.14 ♥ Suppose a random sample of size 9 was obtained from a normal population with mean μ and standard deviation $\sigma = 6.3$. It was determined that the p–value for the test $H_0 : \mu = 80$ versus $H_a : \mu \neq 80$ was 0.8336.

(a) If $\bar{x} > \mu$, find a 95% confidence interval for μ.

(b) Approximately how large of a sample size n would be needed for the margin of error (at 95% confidence) to equal 2.0?

7.15 The resistance (in Ohms) of a certain type of resistor (an electronic device) follows a normal distribution with mean μ and standard deviation σ. A technician randomly selected 25 resistors; the mean resistance was $\bar{x} = 971$ with standard deviation $s = 68$.

(a) Find a 95% confidence interval for μ.

(b) Based upon your answer in 7.15(a), does the mean resistance μ significantly differ from 1,000? Why?

(c) Suppose we wish to test $H_0 : \mu = 1{,}000$ versus $H_a : \mu \neq 1{,}000$ at the $\alpha = 0.05$ significance level. What is the $p-$value for the test?

(d) Based upon your answer in 7.15(c), does the mean resistance μ significantly differ from 1,000? Why?

(e) Suppose the significance level $\alpha = 0.10$. Does the mean resistance μ significantly differ from 1,000? Why?

(f) Suppose the significance level $\alpha = 0.01$. Does the mean resistance μ significantly differ from 1,000? Why?

7.16 ♥ A random sample of size 400 is obtained from a normal population with mean μ and standard deviation $\sigma = 40$. We wish to test $H_0 : \mu = 25$ versus $H_a : \mu < 25$ at the $\alpha = 0.01$ significance level.

(a) For what values of $\bar{x}$ do we reject H_0?

(b) Find the probability of a type II error β against the alternative that $\mu = 15$.

(c) Find the power of the test against the alternative that $\mu = 15$.

(d) Find the power of the test against the alternative that $\mu = 20$.

(e) How could we obtain more power in 7.16(d)?

7.17 A random sample of size 25 is obtained from a normal population with mean μ and standard deviation $\sigma = 20$. We wish to test $H_0 : \mu = 100$ versus $H_a : \mu > 100$ at the $\alpha = 0.05$ significance level. It can be shown that we will reject H_0 if $\bar{X} > 106.58$ (there is no need to verify this). What is the power of this test against the alternative that $\mu = 105.5$?

7.18 A random sample of size 25 is obtained from a normal population with mean μ and standard deviation $\sigma = 20$. We wish to test $H_0 : \mu = 50$ versus $H_a : \mu < 50$ at the $\alpha = 0.05$ significance level. We will reject H_0 if $\bar{X} < 43.42$ (there is no need to verify this). What is the power of this test against the alternative that $\mu = 40$?

7.19 ♥ A random sample of size 49 is obtained from a normal population with mean μ and standard deviation $\sigma = 21$. An inexperienced researcher wishes to test $H_0 : \mu = 10$ versus $H_a : \mu > 10$.

(a) If he simply rejects H_0 when the sample mean $\bar{X} > 12$, what is the probability that the researcher commits a type I error α?

(b) If we want the probability of a type I error to equal 0.01, for what values of $\bar{X}$ should we reject H_0?

7.20 A random sample of size 400 is obtained from a normal population with mean μ and standard deviation $\sigma = 50$. An inexperienced researcher wishes to test $H_0 : \mu = 100$ versus $H_a : \mu < 100$.

(a) The researcher simply decides to reject H_0 if the sample mean $\bar{X} < 95$. What is the probability that she commits a type I error α?

(b) If we want the probability of a type I error to equal 0.05, for what values of $\bar{X}$ should we reject H_0?

7.21 ♥ The discrete random variable X has two possible distributions, p_0 and p_1.

x:	0	1	2
p_0:	0.1	0.2	0.7
p_1:	0.6	0.3	0.1

We observe X one time. We wish to test $H_0 : p_0$ *is correct* vs. $H_a : p_1$ *is correct*. A researcher decides to reject H_0 if $X = 0$ *or* 1 (and will not reject H_0 if $X = 2$).

(a) Determine the probability of a type I error α.

(b) Determine the probability of a type II error β.

(c) Determine the power of the test.

7.22 The discrete random variable X has two possible distributions, p_0 and p_1.

x:	0	2	4	8
p_0:	0.5	0.3	0.1	0.1
p_1:	0.2	0.3	0.3	0.2

We observe X one time. We wish to test $H_0 : p_0$ *is correct* vs. $H_a : p_1$ *is correct*. A researcher decides to reject H_0 if $X > 0$ (and will not reject H_0 if $X = 0$).

(a) Determine the probability of a type I error α.

(b) Determine the probability of a type II error β.

(c) Determine the power of the test.

7.23 ♥ We want to obtain a random sample of size n from a normal population with mean μ and standard deviation $\sigma = 20$. We wish to test $H_0 : \mu = 50$ versus $H_a : \mu < 50$ at the $\alpha = 0.05$ significance level. How large would the sample size n have to be for the power of the test against the alternative $\mu = 40$ to be (at least) 95%?