6.4 Exercises

♥ = answers are provided beginning on page 229.

6.1 ♥ A bowl contains 3 chips: the chips labeled 0, 2, and 4. A chip is randomly selected from the bowl. Let X denote the number printed on the chip. The probability mass function (probability distribution) of X is

$$\begin{array}{rccc} x: & 0 & 2 & 4 \\ P(X=x): & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}$$

(a) Find the mean of X, $\mu = E(X)$.

(b) Find the standard deviation of X, $\sigma = SD(X)$.

(c) Suppose 2 chips are randomly selected from the bowl *with* replacement. Find the sampling distribution of $\bar{X}$.

(d) Determine the mean of $\bar{X}$ using Definition 4.4, i.e. compute $\mu_{\bar{X}} = E(\bar{X}) = \sum_{\bar{x}} \bar{x} P(\bar{X} = \bar{x})$.

(e) Determine the mean of $\bar{X}$ using Theorem 6.1.

(f) Determine the standard deviation of $\bar{X}$ using Definition 4.4, i.e. compute $\sigma_{\bar{X}} = SD(\bar{X}) = \sqrt{\sum_{\bar{x}} \left(\bar{x} - E(\bar{X})\right)^2 P(\bar{X} = \bar{x})}$.

(g) Determine the standard deviation of $\bar{X}$ using Theorem 6.1.

6.2 ♥ Repeat question 6.1(c) assuming the 2 chips are randomly selected *without* replacement.

6.3 A roulette wheel has 38 slots: 18 are black, 18 are red, and 2 are green. A gambler bets that the ball will land in a black slot. He will win \$10 if the ball lands in a black slot, otherwise he will lose \$10. If the random variable X denotes his winnings on any spin of the wheel, the probability distribution of X is clearly

$$\begin{array}{rcc} x: & -10 & 10 \\ P(X=x): & \frac{20}{38} & \frac{18}{38} \end{array}$$

Suppose the gambler plays 20 times. Let $\bar{X}$ denote his mean winnings for the 20 spins.

(a) On average, how much does the gambler expect to win for each play? In other words, find the mean of X, $\mu = E(X)$.

(b) Find the standard deviation of X, $\sigma = SD(X)$.

(c) What is the expected value of his mean winnings for the 20 plays? In other words, find the mean of $\bar{X}$, $\mu_{\bar{X}} = E(\bar{X})$.

(d) Find the standard deviation of his mean winnings $\bar{X}$, $\sigma_{\bar{X}} = SD(\bar{X})$.

(e) On average, how much is his expected *total* winnings (or loss) for the 20 spins?

6.4 ♥ Suppose a bottling plant fills 2-liter soda bottles. The distribution of the amount of soda dispensed into each bottle follows a normal distribution with mean $\mu = 2.02$ liters and standard deviation $\sigma = 0.009$ liters.

(a) Find the probability that a randomly selected bottle contains more than 2.03 liters.

(b) Find the probability that the mean amount of soda $\bar{X}$ in 36 randomly selected bottles is greater than 2.022 liters.

(c) Find the probability that the mean amount of soda $\bar{X}$ in 4 randomly selected bottles is between than 2.010 and 2.015 liters.

(d) Suppose 4 bottles of soda are randomly selected. Determine the 80th percentile of the sample mean $\bar{X}$.

6.5 Suppose the longevity of iPad batteries, X, can be modeled by a normal distribution with mean $\mu = 8.2$ hours and standard deviation $\sigma = 1.2$ hours, i.e. $X \sim N(\mu = 8.2, \sigma = 1.2)$.

(a) Find the probability a randomly selected iPad lasts less than 10 hours.

(b) Suppose 16 iPads are randomly selected. Find the probability that the mean longevity, $\bar{X}$, is less than 7.9 hours.

(c) Find the 25th percentile of the individual battery times, X.

(d) Suppose 16 iPads are randomly selected. Find the 25th percentile of the sample mean $\bar{X}$.

(e) Suppose 16 iPads are randomly selected. Let Y equal the number with longevity more than 10 hours. What is the distribution of Y?

(f) In reference to question 6.5(e), determine the probability that exactly 2 of the iPads last longer than 10 hours.

6.6 The gain, X, of 2N3904 bipolar NPN transistors follow a normal distribution with mean $\mu = 150$ and standard deviation $\sigma = 36$, i.e. $X \sim N(\mu = 150, \sigma = 36)$.

(a) Find the probability that a randomly selected transistor has gain between 150 and 200.

(b) Suppose 9 transistors are randomly selected. Find the probability that the mean gain $\bar{X}$ is more than 183.

(c) Suppose 9 transistors are randomly selected. Find the probability that the mean gain $\bar{X}$ is between 160 and 183.

(d) Suppose 9 transistors are randomly selected. Find the 60th percentile of the sample mean $\bar{X}$.

(e) Suppose 9 transistors are randomly selected. Let Y equal the number of transistors with gain between 150 and 200. What is the distribution of Y?

(f) In reference to question 6.6(e), determine the probability that exactly 3 of the transistors have gain between 150 and 200.

6.7 ♥ Based upon past data, a professor knows that the number of absent students on any given day is strongly skewed to the right with mean $\mu = 8$ and standard deviation $\sigma = 12$.

(a) Suppose 4 days are randomly selected (assume independence). Can you find the probability that the mean number of absences, $\bar{X}$, is less than 2? If so, find the probability. If not, explain why.

(b) Suppose the class meets 32 times during the semester (assume independence). Approximate the probability that the mean number of daily absences during the semester, $\bar{X}$, is less than 7.

(c) Suppose the class meets 32 times during the semester (assume independence). Approximate the probability that the *total* number of absences during the semester is greater than 320. *Hint: Restate this problem in terms of the sample mean $\bar{X}$.*

6.8 The expenditures (in dollars) of customers at a coffee shop has a distribution that is strongly skewed to the right with mean $\mu = 3.50$ and standard deviation $\sigma = 2.00$.

(a) Suppose 12 customers enter the shop (assume independence). Can you find the probability that the mean expenditure, $\bar{X}$, is more than \$3.75? If so, find the probability. If not, explain why.

(b) Suppose 100 customers are randomly selected (assume independence). Approximate the probability that the mean expenditure, $\bar{X}$, is more than \$3.00.

(c) Suppose 100 customers are randomly selected (assume independence). Approximate the probability that the mean expenditure, $\bar{X}$, is between than \$3.00 and \$3.25.

(d) Suppose 100 customers are randomly selected (assume independence). Find the 99th percentile of the sample mean $\bar{X}$.

(e) Suppose 400 customers are randomly selected (assume independence). Find the probability that the *total* expenditure is less than \$1,360. *Hint: Restate this problem in terms of the sample mean $\bar{X}$.*

6.9 Packages of batteries contain 2 batteries each. Suppose the number of good batteries in a package, X, has the following probability distribution.

x:	0	1	2
$P(X = x)$:	0.1	0.1	0.8

(a) Find the mean and standard deviation of X.

(b) Suppose we randomly select 100 packages of batteries (assume independence). Determine the probability distribution for the mean number of good batteries per package, $\bar{X}$. *Be sure to state all parameters.*

(c) Suppose we randomly select 100 packages of batteries (assume independence). Approximate the probability that the mean number of good batteries per package, $\bar{X}$, is less than 1.6.

(d) Suppose we randomly select 100 packages of batteries (assume independence). Find the 95th percentile of the sample mean $\bar{X}$.

(e) Suppose we randomly select 100 packages of batteries (assume independence). Approximate the probability that the *total* number of good batteries is 155 or less. *Hint: Restate this problem in terms of the sample mean $\bar{X}$.*

6.10 The number of applications a vendor sells per day on Google Play, X, can be modeled by a Poisson distribution with mean $\mu = 16$, i.e. $X \sim Pois(\mu = 16)$.

(a) Find the mean and standard deviation of X.

(b) Suppose the vendor records the daily sales over a 30 day period. If $\bar{X}$ equals the mean number of daily sales over this 30 day period, determine the approximate distribution of $\bar{X}$. *Be sure to state all parameters.*

(c) Suppose the vendor records their sales over a 30 day period. If $\bar{X}$ equals the mean number of daily sales over this 30 day period, determine the probability that $\bar{X}$ is greater than 15.

(d) Suppose the vendor records their sales over a 30 day period. Approximate the 95th percentile of the mean daily sales $\bar{X}$.

(e) Approximate the probability that the vendor sells more than 6000 applications in 1 year. *Hint: Restate this problem in terms of the sample mean* $\bar{X}$*.*

6.11 ♥ In reference to question 6.6, how large must the sample size n be for the sample mean gain $\bar{X}$ to fall within 5 of $\mu = 150$ with 90% probability? In other words, find n such that $0.90 = P(145 < \bar{X} < 155)$.

6.12 In reference to question 6.8, how many customers n must be randomly selected for the sample mean expenditure $\bar{X}$ to fall within 0.10 of $\mu = 3.50$ with 80% probability? In other words, find n such that $0.80 = P(3.4 < \bar{X} < 3.6)$. *Because the distribution of X is not normal, assume that n will exceed 30 (so the CLT can give us normality). We could not do this problem if $n < 30$.*