Homework (Multiple Regression)
Biostatistics (STAT:3510; Bognar)

1. A new speech processor for existing cochlear implants has been developed. A clinical trial involving 23 subjects compared the new processor to the old processor. The following multiple regression model was used:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$$

where y denotes the speech recognition score (i.e. y is simply the percentage of spoken words correctly understood), x_1 denotes the subjects age, and x_2 equals 0 if the subject is using the old processor, and 1 if using the new processor. The population regression equation is

$$\mu_{y|x_1,x_2} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

The analysis yielded the following results in R:

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Coefficients:
           Estimate Std. Error t value Pr(>|t|)
Intercept   43.1376    12.6781   3.403   0.0028
age         -0.1034     0.8763  -0.118   0.9072
processor   22.3329     8.1293   2.747   0.0124
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Note that $\hat{\beta}_0 = 43.1376$, $\widehat{se}(\hat{\beta}_0) = 12.6781$, $\hat{\beta}_1 = -0.1034$, $\widehat{se}(\hat{\beta}_1) = 0.8763$, $\hat{\beta}_2 = 22.3329$, and $\widehat{se}(\hat{\beta}_2) = 8.1293$.

(a) Does age significantly influence the speech recognition score? To answer this, test $H_0 : \beta_1 = 0$ vs $H_a : \beta_1 \neq 0$ at the $\alpha = 0.05$ significance level (find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion).

(b) Find the $p-$value for the test in part (a). Does age significantly affect speech recognition score? Why?

(c) Find a 95% confidence interval for β_1. Is age significant? Why?

(d) Find a 95% confidence interval for β_2. Is there a significant difference between the old and new processors? Why?

(e) If we were to test $H_0 : \beta_2 = 0$ vs $H_a : \beta_2 \neq 0$ at the $\alpha = 0.05$ significance level, would the $p-$value be less than 0.05 or more than 0.05? Why? Base your answer on the CI in part (d).

(f) Find the $p-$value for the test in part (e). Is there a significant difference between the old and new processors? Why?

(g) Approximate the mean speech recognition score for 33 year olds with the new processor, i.e. approximate $\mu_{y|x_1=33,x_2=1}$.

(h) In part (g), R determined that $\widehat{se}(\hat{y}) = 5.77$. Compute a 95% confidence interval for $\mu_{y|x_1=33,x_2=1}$.

(i) Is the mean speech recognition score for 33 year olds with the new processor significantly higher than 50? Why?