Homework
Biostatistics (STAT:3510; Bognar)

1. In the Iowa Driving Simulator, the number of times the center line is crossed by individuals that are under the influence of alcohol has a distribution that is skewed to the right with mean μ and standard deviation $\sigma = 7$. For the 49 participants that drove after drinking alcohol, the mean number of times the center line was crossed was $\bar{x} = 10$. Suppose we wish to perform the one-sided test $H_0 : \mu = 9$ versus $H_a : \mu > 9$ at the $\alpha = 0.01$ significance level.

   (a) Perform this test. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

   (b) Based upon your answer in (1a), will the $p-$value for the test be less than α or greater than α? Why?

   (c) Find the $p-$value for the test in (1a).

   (d) Based upon your answer in (1c), is the mean number of crossings μ significantly higher than 9? Why?

2. Suppose the weight of bags of M&M's, X (in ounces), follow a normal distribution with mean μ ounces and standard deviation $\sigma = 0.10$ ounces, i.e. $X \sim N(\mu, \sigma = 0.10)$. A random sample of 4 bags had an average weight $\bar{x} = 15.9$ ounces. Suppose we wish to test $H_0 : \mu = 16$ vs $H_a : \mu < 16$ at the $\alpha = 0.05$ significance level.

   (a) What is the $p-$value for this test?

   (b) Is the mean weight μ significantly less than 16 ounces? Why?

   (c) Suppose the significance level $\alpha = 0.01$. Is the mean weight μ significantly less than 16 ounces? Why?

   (d) Could we perform the above analysis if the weights did *not* have a normal distribution? Why?

3. The amount of time per day, X (in hours), office workers spend working on a computer can be modeled by a normal distribution with mean μ and standard deviation σ, i.e. $X \sim N(\mu, \sigma)$. A manager wants to infer about the population mean μ, so he randomly selects 5 employees and observes their computer time over the course of a day. The raw data is:

   $$6.5,\ 7.1,\ 5.9,\ 6.2,\ 6.3$$

   Hint: $n = 5,\ \bar{x} = 6.4,\ s = 0.4472.$

   (a) Test $H_0 : \mu = 6$ vs. $H_a : \mu \neq 6$ at the $\alpha = 0.01$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

   (b) Based upon your answer in (3a), does the population mean computer time μ significantly differ from 6 hours? Why?

   (c) Find the $p-$value for the test in (3a).

   (d) Based upon your answer in (3c), does the population mean computer time μ significantly differ from 6 hours? Why?

   (e) Find a 99% confidence interval for μ.

   (f) Based upon your answer in (3e), does the population mean computer time μ significantly differ from 6 hours? Why?

   (g) Another manager wants to do the one-sided test $H_0 : \mu = 6.8$ vs. $H_a : \mu < 6.8$ at the $\alpha = 0.10$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

   (h) Could we perform the above analysis if the computer times did *not* have a normal distribution? Why?

4. Wood et. al (1988) studied the efficacy of diet for losing weight. The study, which lasted one year, involved only men. The weight loss for dieting men follows a normal distribution with mean μ and standard deviation σ. A group of $n = 16$ dieting men lost an average of $\bar{x} = 7.2$ pounds with standard deviation $s = 4.4$ pounds.

   (a) Find a 90% confidence interval for μ.

   (b) Test $H_0 : \mu = 5.5$ vs. $H_a : \mu \neq 5.5$ at the $\alpha = 0.10$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

   (c) Approximate the $p-$value for the test in (4b).

(d) Based upon your answer in (4c), does the population mean weight loss μ significantly differ from 5.5 pounds? Why?

(e) Test $H_0 : \mu = 5.5$ vs. $H_a : \mu > 5.5$ at the $\alpha = 0.10$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(f) Approximate the $p-$value for the test in (4e).

(g) Based upon your answer in (4f), is the population mean weight loss μ significantly more than 5.5 pounds? Why?

(h) Could we perform the above analysis if weight loss did *not* have a normal distribution? Why?