

**HOMEWORK**  
**BIOSTATISTICS (STAT:3510; BOGNAR)**

- Suppose you roll a standard 6-sided die. If you roll a “1” (1), you randomly select one chip from a bowl containing 2 red ( $R$ ) and 3 white ( $W$ ) chips. If you don’t roll a “1” ( $1^c$ ), you randomly select 1 chip from a bowl containing 7 red ( $R$ ) and 3 white ( $W$ ) chips.
  - Find the probability that you roll a “1” *and* obtain a white chip ( $W$ ).
  - Determine the probability you obtain a white chip, i.e. find  $P(W)$ .
  - Given that a white chip was obtained, determine the probability that a 1 was rolled on the die, i.e. find  $P(1|W)$ .
  - Determine the probability that a 1 is rolled on the die *or* a red chip is selected from the bowl, i.e. find  $P(1 \cup R)$ .
- The probability that a passenger will attempt to board an airplane with illegal drugs is 0.005 (i.e.  $P(D) = 0.005$ ). Given that a passenger has illegal drugs, the probability that the alarm will sound is 0.97 (i.e.  $P(A|D) = 0.97$ ). If a passenger does not have illegal drugs, the probability that the alarm will not sound is 0.95 (i.e.  $P(A^c|D^c) = 0.95$ ).
  - What is the sensitivity of the drug detection machine?
  - What is the specificity of the drug detection machine?
  - Find the probability that the alarm does not sound given that the passenger is carrying drugs (i.e. find  $P(A^c|D)$ ).
  - Suppose a passenger is randomly selected. Find the probability that the alarm sounds when he/she enters security (i.e. find  $P(A)$ ).
  - Given that the alarm sounds, find the probability that the passenger actually has illegal drugs. (i.e. find  $P(D|A)$ ). This quantity is known as the “predictive value of a positive test”.
  - Find the “predictive value of a negative test” (i.e. find  $P(D^c|A^c)$ ). In words, what does this quantity mean?
- A farm has two types of trees: 30% are orange trees ( $O$ ) and 70% are apple trees ( $A$ ). Frost ( $F$ ) has damaged 40% of the orange trees (i.e.  $P(F|O) = 0.40$ ) and 10% of the apple trees.
  - Find the probability that a randomly selected tree was damaged by frost *and* is an apple tree.
  - Find the probability that a randomly selected tree has been damaged by frost.
  - Given that a randomly selected tree has been damaged by frost, determine the probability that it is an apple tree.
- A basket contains 4 puppies: one of the puppies has 1 spot, one of the puppies has 2 spots, and the remaining two puppies have 4 spots. Suppose *two* puppies are selected at random *without* replacement. Let the random variable  $X$  equal the *total* number of spots on the selected puppies.
  - Find the probability distribution of  $X$ .
  - Find the probability that the puppies have a total of 5 spots, i.e. find  $P(X = 5)$ .
  - Find the probability that the puppies have a total of 6 or more spots, i.e. find  $P(X \geq 6)$ .
  - Find the probability that the puppies have 5 or fewer spots *or* 8 spots, i.e. find  $P(X \leq 5 \cup X = 8)$ . *These events are mutually exclusive.*
  - On average, how many spots do we expect on the two selected puppies? In other words, find  $\mu = E(X)$ .
  - Compute  $\sigma^2 = Var(X)$ .
- Suppose a bowl has 5 chips; two chips are labeled “2”, and three chips are labeled “3”. Suppose *two* chips are selected at random *with* replacement. Let the random variable  $X$  equal the *product* of the two draws (e.g. if the first draw is a 2 ( $2_1$ ) and the second draw is a 3 ( $3_2$ ), then the product is  $2 \times 3 = 6$ ).
  - Find the probability distribution of  $X$ .
  - Find the probability that the *product* of the two draws is less than or equal to 6, i.e. find  $P(X \leq 6)$ .
  - Compute the expected value of  $X$ ,  $\mu = E(X)$ .
  - Compute  $\sigma = SD(X)$ .