Homework
Prob. and Stat. for Eng. (STAT:2020; Bognar)

1. Suppose the random variable X has the following probability distribution. Find the following probabilities.

$x:$	0	1	2	3	4
$f(x) = P(X = x):$	0.2	0.1	0.2	0.2	0.3

   (a) $P(X \leq 2)$
   (b) $P(X < 2)$
   (c) $P(X \leq 2 \cap X \geq 1)$
   (d) $P(X = 1 \cup X < 3)$
   (e) $P(X = 2 | X \leq 2)$

2. A large warehouse contains 2-packs, 4-packs, and 8-packs of batteries. Suppose the random variable X equals the number of batteries in a randomly selected package of batteries. It is known that X has probability distribution

$$f(x) = P(X = x) = \frac{8}{7x} \quad \text{for } x = 2, 4, 8$$

   (a) What is $P(X = 2)$?
   (b) Determine $P(X \geq 4)$.
   (c) Find the cumulative distribution function of X, $F(x)$. Be sure to define the cdf for all $x \in (-\infty, \infty)$.

3. Suppose the discrete random variable X has probability distribution

$$f(x) = P(X = x) = \frac{1}{2^x} \quad \text{for } x = 1, 2, \ldots$$

   (a) Find $P(X = 5)$.
   (b) Determine $P(X \geq 2)$.
   (c) Find $P(X \leq 4 \cap X \geq 4)$.
   (d) Find $P(X \leq 4 \cup X \geq 4)$.
   (e) Determine $P(X \leq 3 | X \geq 2)$.

4. A basket contains 4 puppies: one of the puppies has 1 spot, one of the puppies has 2 spots, and the remaining two puppies have 4 spots. Suppose *two* puppies are selected at random *without* replacement. Let the random variable X equal the *total* number of spots on the selected puppies.

   (a) Find the probability distribution of X.
   (b) Find the probability that the puppies have a total of 5 spots, i.e. find $P(X = 5)$.
   (c) Find the probability that the puppies have a total of 6 or more spots, i.e. find $P(X \geq 6)$.
   (d) Find the cumulative distribution function of X, $F(x)$. Be sure to define the cdf for all $x \in (-\infty, \infty)$.

5. Suppose a bowl has 9 chips; one chip is labeled "1", three chips are labeled "3", and five chips are labeled "5". Suppose *two* chips are selected at random *with* replacement. Let the random variable X equal the *absolute difference* between the two draws (e.g. if the first draw is a 1 (1_1) and the second draw is a 5 (5_2), then the absolute difference is $|1 - 5| = 4$).

   (a) Find the probability distribution of X.
   (b) Use the probability distribution to find the probability that both draws are the same.
   (c) Use the probability distribution to find the probability that both draws are *not* the same.
   (d) Find the cumulative distribution function of X, $F(x)$. Be sure to define the cdf for all $x \in (-\infty, \infty)$.