

**HOMEWORK**  
**PROB. AND STAT. FOR ENG. (STAT:2020; BOGNAR)**

1. A bowl contains 3 chips: the chips labeled 0, 2, and 4. A chip is randomly selected from the bowl. Let  $X$  denote the number printed on the chip. The probability mass function (probability distribution) of  $X$  is

$$\begin{array}{rcc} x : & 0 & 2 & 4 \\ P(X = x) : & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}$$

- (a) Find the mean of  $X$ , i.e. find  $\mu = E(X) = \sum_x xP(X = x)$ .
- (b) Find the standard deviation of  $X$ , i.e. find  $\sigma = SD(X) = \sqrt{\sum_x (x - \mu)^2 P(X = x)}$ .
- (c) Suppose 2 chips are randomly selected from the bowl *with* replacement. Find the sampling distribution of  $\bar{X}$ .
- (d) Suppose 2 chips are randomly selected from the bowl *with* replacement. Find  $P(\bar{X} \leq 1)$ .
- (e) Determine the mean of  $\bar{X}$ , i.e. compute  $\mu_{\bar{X}} = E(\bar{X}) = \sum_{\bar{x}} \bar{x}P(\bar{X} = \bar{x})$ .
- (f) According to the theorem given in class, the mean of  $\bar{X}$  is  $\mu_{\bar{X}} = E(\bar{X}) = \mu$ . Does this hold true when you compare parts (1e) and (1a)?
- (g) Determine the standard deviation of  $\bar{X}$ , i.e. compute  $\sigma_{\bar{X}} = SD(\bar{X}) = \sqrt{\sum_{\bar{x}} (\bar{x} - \mu_{\bar{X}})^2 P(\bar{X} = \bar{x})}$ .
- (h) According to the theorem given in class, the standard deviation of  $\bar{X}$  is  $\sigma_{\bar{X}} = SD(\bar{X}) = \sigma/\sqrt{n}$ . Compute  $\sigma/\sqrt{n}$  (remember, we derived  $\sigma$  in part (1b)). Does this equal the result from part (1g)?
- (i) Suppose 100 chips are randomly selected from the bowl with replacement. Let  $\bar{X}$  denote the mean of the selected chips. Find what is the approximate distribution of  $\bar{X}$ ?
- (j) Suppose 100 chips are randomly selected from the bowl with replacement. Let  $\bar{X}$  denote the mean of the selected chips. Approximate the probability that  $\bar{X}$  is between 2.1 and 2.4.