(a) Find the probability using the binomial distribution.

We are in a "binomial setting" and thus $X \sim Bin(n = 200, p = 0.7)$. *Now,*

$$P(X \geq 150) = \sum_{x=150}^{200} \binom{n}{x} p^x (1-p)^{n-x} = \sum_{x=150}^{200} \binom{200}{x} 0.7^x 0.3^{200-x}$$

which will be a lot of work to evaluate without a computer. In addition, depending upon your computer and operating system, the combinations and exponentials may cause numerical over/underflows and produce computational errors (especially if n *were larger than 200). Using the binomial applet at*

`http://www.stat.uiowa.edu/~mbognar/applets/bin.html`

we see that the actual probability is 0.0695 *(using the complement rule).*

(b) Approximate the probability using the normal approximation to the binomial distribution.

Because $n \geq 30$, $np \geq 5$, *and* $n(1-p) \geq 5$, *then we can approximate a* $Bin(n = 200, p = 0.7)$ *distribution by*

$$Y \sim N(\mu = np = 140, \sigma = \sqrt{np(1-p)} = 6.48).$$

Therefore,

$$P(X \geq 150) \simeq P(Y > 149.5) = P\left(\frac{Y - 140}{6.48} > \frac{149.5 - 140}{6.48}\right) = P(Z > 1.47) = 0.0708$$

Finding $P(Y > 149.5)$ *instead of* $P(Y > 150)$ *is known as a* continuity correction. *The continuity correction yields more accurate approximations. We can similarly approximate* $P(X \leq 150)$ *by* $P(Y < 150.5)$ *and* $P(X = 150)$ *by* $P(149.5 < Y < 150.5)$.

(c) Using the applet at

`http://www.stat.uiowa.edu/~mbognar/applets/binnormal.html`

find $P(X \leq 125)$ using the binomial distribution and also approximate this probability via the normal approximation (i.e. find $P(Y \leq 125.5)$).

Enter 200 in the n *box, 0.7 in the* p *box, 125 in the* x *box, and select* $P(X \leq x)$ *from the drop-down box. The binomial probability,* 0.01383, *is shown in the pink box. Checking the normal approximation check-box plots the normal approximation and determines* $P(Y \leq 125.5) = 0.01263$ *(the applet automatically accounts for the continuity correction).*

5.4 Exercises

♥ = answers are provided beginning on page 169.

5.1 Suppose $X \sim Unif(5, 10)$.

(a) Make a graph of the pdf of X.

(b) Find $P(5 < X < 6)$.

(c) Find $P(5 \leq X \leq 6)$.

(d) Determine $P(6 < X < 8)$.

(e) Find $P(X = 7)$.

(f) Determine $P(X < 5)$.

(g) Determine the 90th percentile of X, i.e. find the value c such that $P(X < c) = 0.90$.

5.2 ♥ Suppose $X \sim N(\mu = 40, \sigma = 3)$.

(a) Make a graph of the distribution of X. Be sure to clearly mark the location of the mean and points of inflection on the horizontal axis.

(b) Use the empirical rule to find $P(37 < X < 43)$.

(c) Use the empirical rule to find $P(X > 43)$.

(d) Use the empirical rule to find $P(X > 34)$.

(e) Use the empirical rule to find $P(34 < X < 43)$.

(f) Use the empirical rule to find $P(43 < X < 46)$.

5.3 ♥ Recall that the standard normal distribution $Z \sim N(\mu = 0, \sigma = 1)$. Use the standard normal table to answer the following.

(a) Determine $P(Z > 0.68)$.

(b) Determine $P(0.82 < Z < 1.17)$.

(c) Find the value c such that $P(Z < c) = 0.20$.

(d) Find the value c such that $P(Z > c) = 0.025$.

(e) Find the value c such that $P(-c < Z < c) = 0.90$.

5.4 Scores on the ACT exam have a $N(\mu = 18, \sigma = 6)$ distribution, while scores on the SAT exam have a $N(\mu = 500, \sigma = 100)$ distribution. Suppose Xiaoyu got a 27 on the ACT exam, while Emily got a 720 on the SAT exam. Who has the higher relative score? Why?

5.5 A *nit* is a measure of brightness (one nit is equal to one candela per square meter). The LCD screens from a manufacturer have brightness, X, that follows a normal distribution with mean $\mu = 250$ nits and standard deviation $\sigma = 15$ nits, i.e. $X \sim N(250, 15)$.

(a) Find the probability that a randomly selected screen has brightness between 250 and 270 nits.

(b) Suppose screens with brightness levels in the highest 25% are sold as "premium" panels. How many nits must a screen produce to be sold as premium?

5.6 ♥ Suppose the length of time an iPad battery lasts, X, can be modeled by a normal distribution with mean $\mu = 8.2$ hours and standard deviation $\sigma = 1.2$ hours, i.e. $X \sim N(\mu = 8.2, \sigma = 1.2)$.

(a) Find the probability that a randomly selected iPad lasts longer than 10 hours.

(b) Find the probability that a randomly selected iPad lasts between 7 and 10 hours.

(c) What is the 3rd percentile of the battery times?

5.7 Suppose the length of the US \$20 bill is normally distributed with mean $\mu = 6.140$ inches with standard deviation $\sigma = 0.002$ inches.

(a) Find the probability that the length of a randomly selected \$20 bill is less than 6.145 inches.

(b) Find the 60^{th} percentile of the bill lengths.

(c) Given that the length of a randomly selected \$20 bill is more than 6.140 inches, determine the probability that it is less than 6.141 inches.

5.8 ♥ After being cut by a special saw, suppose the length of aluminum alloy billets, X, is normally distributed with mean $\mu = 96.2$ inches and standard deviation $\sigma = 0.2$ inches.

(a) If a billet is randomly selected, determine the probability that it is longer than 96.55 inches.

(b) Determine the inter-quartile range (IQR) of the billet lengths.

(c) Given that a billet is longer than 96.2 inches, determine the probability that it is longer than 96.3 inches.

5.9 A chocolate manufacturer produces boxes of chocolates whose weights, X, follow a normal distribution with mean $\mu = 16.2$ ounces and standard deviation $\sigma = 0.1$ ounces.

(a) Find the probability that a randomly selected box contains less than 16 ounces of chocolate.

(b) Suppose the lightest 38.21% of boxes are sent to discount retailers. To be sent to a discounter, what is the most a box of chocolates can weigh?

(c) What is the interquartile range (IQR) of the weights?

5.10 Suppose $X \sim N(\mu, \sigma)$. If $P(X < 100) = 0.16$ and $P(X > 142) = 0.5$, use the empirical rule to find μ and σ.

5.11 Suppose $X \sim N(\mu, \sigma)$. If $P(X > 50) = 0.10$ and $P(X < 60) = 0.97$, find μ and σ. *Hint: You will get 2 equations and 2 unknowns; solve for μ and σ.*

5.12 Suppose the weight, X, of bags of oranges follow a normal distribution with mean μ pounds and standard deviation $\sigma = 0.2$ pounds. If the 10th percentile of the weights is 5.05 pounds, find μ.

5.13 ♥ Bags of coffee have weights, X, that follow a normal distribution with mean $\mu = 1.03$ pounds and standard deviation σ pounds. If 97% of the bags weigh more than 1.00 pounds, find σ.

5.14 Suppose the length of time, X, the battery lasts on Google's Nexus 7 tablet can be modeled by a normal distribution with mean $\mu = 7.7$ hours and standard deviation σ hours. If 11.7% of the tablets last longer than 8.5 hours, find $\sigma^2 = Var(X)$.

5.15 After being cut by a special saw, suppose the length of aluminum alloy billets, X, is normally distributed with mean $\mu = 96.2$ inches and standard deviation $\sigma = 0.2$ inches. Suppose you randomly select 20 billets (assume independence).

(a) Determine the probability that exactly 2 are longer than 96.5 inches. *Hint: First find the probability that a single billet is longer than* 96.5 *inches, then use the binomial distribution.*

(b) Determine the probability that 19 or fewer are have a length less than 96.5 inches.

(c) Determine the probability that all 20 billets are between 96.0 and 96.5 inches.

5.16 ♥ It is known that 20% of all credit applicants have poor credit ratings. Suppose 50 applicants are randomly selected (assume independence).

(a) Use the binomial distribution to find the probability that 3 or less have poor credit.

(b) Use the normal approximation to find the probability that 3 or less have poor credit.

(c) Use the binomial distribution to find the probability that exactly 13 have poor credit.

(d) Use the normal approximation to find the probability that exactly 13 have poor credit.

(e) Verify your answers in parts (a)-(d) using the applet at

`http://www.stat.uiowa.edu/~mbognar/applets/binnormal.html`

5.17 The labor force participation rate is approximately 63% in the US (i.e. approximately 63% of eligible adults actually work). Suppose 100 adults are randomly selected (assume independence).

(a) Use the binomial distribution to find the probability that exactly 70 work.

(b) Use the normal approximation to find the probability that exactly 70 work.

(c) Use the normal approximation to find the probability that 70 or less work.

(d) Verify your answers in parts (a)-(c) using the applet at

`http://www.stat.uiowa.edu/~mbognar/applets/binnormal.html`