Homework 13 **Name:** _____
Elementary Statistics & Inference
STAT:1020, Bognar

Print this pdf file, show your work in the provided space, use scanning app to scan pages (in order) into a single pdf file, submit in Gradescope. Be sure to get entire page in each shot — keep pages flat when scanning. You can use an iPad/tablet too.

1. Let p denote the population proportion of (budget) LCD panels that have defective pixels. A researcher randomly selected 100 LCD panels and determined that 8 have defective pixels.

   (a) Suppose we wish to test $H_0 : p = 0.12$ versus $H_a : p \neq 0.12$ at the $\alpha = 0.05$ significance level using the score test. Approximate the $p-$value for this test.

   (b) In reference to (1a), does the proportion of defective panels significantly differ from 0.12? Why?

   (c) Find an approximate 95% Wald confidence interval for p.

   (d) What is the estimated standard error of $\hat{p}$, $\widehat{se}(\hat{p})$?

   (e) What is the estimated margin of error (at 95% confidence)?

   (f) How many panels would need to be selected for the margin of error (at 95% confidence) to equal 0.03?

2. Suppose the population proportion of shoulder surgery patients that can complete a certain physical task (one week after surgery) is p. To infer about p, a researcher randomly selected 60 shoulder surgery patients and determined that 6 could complete the physical task (one week after surgery).

   (a) Find an approximate 95% Wald confidence interval for p.

   (b) Find an approximate 95% Agresti-Coull confidence interval for p.

   (c) Based upon your answer in (2b), does p significantly differ from 0.15? Why?

   (d) Based upon your answer in (2b), does p significantly differ from 0.50? Why?

   (e) Suppose we wish to test $H_0 : p = 0.50$ versus $H_a : p \neq 0.50$ at the $\alpha = 0.05$ significance level. Based upon your answer in (2b), will the $p-$value for the test be less than or greater than α? Why?

3. A retailer is seeking to study the satisfaction of its customers. The population proportion of satisfied customers is p. A random sample of 100 customers yielded 90 that were satisfied. We would like to determine if more than 80% of the customers are satisfied.

   (a) Test $H_0 : p = 0.8$ versus $H_a : p > 0.8$ at the $\alpha = 0.05$ significance level using the score test. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Find the $p-$value for the test in (3a).

(c) Based upon your answer in (3b), is the proportion of satisfied customers significantly higher than 0.80? Why?

4. After smoking marijuana, 7 out of 100 subjects failed a driving test on the Iowa Driving Simulator. Only 7 out of 140 subjects *not* under the influence of marijuana failed the test. Let p_1 denote the population proportion of marijuana users that fail the test, and let p_2 denote the population proportion of drivers *not* under the influence of marijuana that fail the driving test.

   (a) Find an 80% confidence interval for $p_1 - p_2$. Is there a significant difference between the groups? Why?

   (b) Suppose we wish to test $H_0 : p_1 = p_2$ versus $H_a : p_1 \neq p_2$. Find the $p-$value for this test. Is there a significant difference in the proportion of marijuana users and non-marijuana users fail the test? Why?

5. A high-quality diamond cutter can receive raw diamonds from two mines: one is in Russia and the other is in Africa. The Russian mine charges slightly more for its diamonds. However, if the Russian mine has a higher proportion of "colorless" diamonds, the diamond cutter will choose to buy his diamonds from them. Let p_1 denote the proportion of colorless diamonds from the Russian mine, and let p_2 denote the proportion of colorless diamonds from the African mine. A random sample of 50 diamonds from each mine found the following:

$$\begin{aligned} \text{Russia:} \quad & n_1 = 50, \text{ number of colorless} = 10 \\ \text{Africa:} \quad & n_2 = 50, \text{ number of colorless} = 6 \end{aligned}$$

(a) Test $H_0 : p_1 = p_2$ versus $H_a : p_1 > p_2$ at the $\alpha = 0.05$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Find the $p-$value for the test in (5a).

(c) Based upon your analysis, is it worthwhile for the diamond cutter to pay extra for diamonds from the Russian mine? Why?