22M174/22C174: Optimization techniques.

Homework 10. Due 05/08/13.

1. Consider the matrix $A \in \mathbb{R}^{4\times 2}$

$$A = \begin{bmatrix} 1 & 4 \\ -3 & 2 \\ 12 & -2 \\ -10 & 6 \end{bmatrix}.$$

Solve the linear least squares problem $\min_{x\in\mathbb{R}^2} ||b - Ax||_2$ for the vector $b \in \mathbb{R}^4$

$$b = \begin{bmatrix} 10 \\ 2 \\ 24 \\ 0 \end{bmatrix}$$

using the normal equations.

2. Let $A \in \mathbb{R}^{m\times n}$. Show that for $p > 0 \in \mathbb{R}$ the matrix

$$A^T A + pI_n$$

is nonsingular. Then show that the pseudo-inverse $A^+ \in \mathbb{R}^{n\times m}$ of A satisfies

$$A^+ = \lim_{p\to 0^+} (A^T A + pI_n)^{-1} A^T.$$

3. Let $F : \mathbb{R}^n \to \mathbb{R}^n$ and $x^* \in \mathbb{R}^n$.

   (a) Show that if $F(x^*) = 0 \Rightarrow x^*$ is a global minimizer of $\|F(x)\|$.

   (b) Show by a counterexample that a global minimizer x^* of $\|F(x)\|$ is not necessarily a zero of $F(x)$.