22M174/22C174: Optimization techniques.

Homework 1. Due 02/04/13.

1. Prove Theorem I.2.1.1.

2. Prove Lemma I.2.2.9.

3. Find the stationary points of the three following objective functions

$$\begin{aligned}
f_1(x_1, x_2) &= 2\cos(x_1)x_2 + x_2^2, \\
f_2(x_1, x_2) &= 3x_1^2 + 2x_2^4 - 8x_2^2 - 10, \\
f_3(x_1, x_2) &= 12x_1x_2(x_1 - x_2 + 1) + 4x_2^3 - 6x_2^2.
\end{aligned}$$

   Which of these points are local minimizers, local maximizers, or saddle points?

4. Is $[0, 0, 0]^T$ a local minimizer of $f(x_1, x_2, x_3) = 4x_1^2 + 2x_3\cos(3x_2)$? If not, find points arbitrarily close to $[0, 0, 0]^T$ with a lower function value.

5. Show that $[0, 0]^T$ is a local minimizer of $f(x_1, x_2) = (x_1 - 2x_2^2)(x_1 - 8x_2^2)$ along every line passing through $[0, 0]^T$. Show also that $f(3x_2^2, x_2) < f(0, 0)$ if $x_2 \neq 0$.