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Jorgensen, Palle E. T. (1-IA); **Pedersen, Steen** (1-WRTS)

Dense analytic subspaces in fractal L^2-spaces. (English. English summary)

J. Anal. Math. **75** (1998), 185–228.

The authors consider fractal measures μ supported on compact sets in $\mathbf{R}^\nu$, which arise from iteration algorithms that are defined from a given affine system of maps in $\mathbf{R}^\nu$. Then, they study the possible orthogonal harmonic function systems in $L^2(\mu)$, and, in particular, they give conditions for $L^2(\mu)$ to have an orthonormal harmonic basis.

The measure μ is defined as the unique solution satisfying the equation

$$\int f\,d\mu = \frac{1}{N}\sum_{b\in B}\int f(\sigma_b(x))\,d\mu(x),$$

for all continuous functions f, where $B\subset\mathbf{R}^\nu$ is a set of N elements, $\sigma_b(x)=R^{-1}x+b$, with R a real matrix with eigenvalues $|\xi|>1$ and satisfying an open-set condition: there exists a bounded open set V such that $\bigcup_{b\in B}\sigma_b V\subset V$, and $\sigma_b V\cap\sigma_{b'}V=\varnothing$ if $b\neq b'$.

For the case $\nu=1$, $R=1$ and $B=\{0,1/2\}$, they are able to show that $\{\mathrm{e}^{i2\pi nx}\colon\ n=0,1,4,5,16,17,20,21,\cdots\}$ is an orthonormal basis in $L^2(\mu)$. Extensions to higher dimensions are also proved under additional assumptions on the measure μ. Further examples are given in $\mathbf{R}^3$ to the case of the Eiffel Tower.

The proofs of the main results are based on the transfer operator of Ruelle. An alternative approach to getting the basis properties of the exponentials has also been considered in the paper by Robert S. Strichartz [J. Anal. Math. 75 (1998), 229–231; MR 2000a:46046; see the following review]. *Javier Soria* (E-BARU-AM)

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Strichartz, Robert S. (1-CRNL)

Remarks on: "Dense analytic subspaces in fractal L^2-spaces" [J. Anal. Math. 75 (1998), 185–228; MR 2000a:46045] by P. E. T. Jorgensen and S. Pedersen. (English. English summary)

J. Anal. Math. **75** (1998), 229–231.

The author gives an alternative method for proving completeness of the bases of exponentials in $L^2(\mu)$, which was first considered in the paper cited in the heading (see the preceding review). The idea is reminiscent of the Albert Cohen criterion in wavelet theory.

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