τ-TILTING FINITE ALGEBRAS

A finite dimensional algebra A is **τ-tilting finite** if there are only finitely many finite dimensional silting A-modules, up to equivalence.

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- every torsion class in $\operatorname{Mod} A$ is of the form $\operatorname{Gen} T$ for a finite dimensional silting module T
- the map α induces a bijection

$$\left\{ \begin{array}{c} \text{fin. dim.} \\ \text{silting } A\text{-modules} \end{array} \right\} / \sim \longrightarrow \left\{ \begin{array}{c} \text{pseudoflat} \\ \text{ring epis } A \to B \end{array} \right\} / \sim$$

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Theorem (A-Marks-Vitória). The following are equivalent for a finite dimensional algebra A.

(i) A is τ-tilting finite.

(ii) All silting A-modules are finite dimensional, up to equivalence.

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(i) A is τ-tilting finite.

(ii) All silting A-modules are finite dimensional, up to equivalence.

(iii) There are only finitely many pseudoflat ring epimorphisms with domain A, up to equivalence.

Hereditary algebras

Theorem (A-Marks-Vitória).
If A is a finite dimensional hereditary algebra, there is a bijection

$$\left\{ \begin{array}{c} \text{minimal} \\ \text{silting } A\text{-modules} \end{array} \right\} / \sim \;\longrightarrow\; \left\{ \begin{array}{c} \text{homological} \\ \text{ring epis } A \to B \end{array} \right\} / \sim$$

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The inverse map:

$$T = B \oplus \operatorname{Coker} \lambda \qquad \leftarrow\!\shortmid \qquad \lambda : A \to B$$

t-STRUCTURES

A pair of classes $(\mathcal{U}, \mathcal{V})$ in $D(A)$ is a *torsion pair* if

- $\mathcal{U}$ and $\mathcal{V}$ are closed under direct summands,
- $\operatorname{Hom}_{D(A)}(\mathcal{U}, \mathcal{V}) = 0$, and
- for any object $X \in D(A)$ there is a triangle

$$U \to X \to V \to U[1]$$

  in $D(A)$ with $U \in \mathcal{U}$ and $V \in \mathcal{V}$.

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A torsion pair $(\mathcal{U}, \mathcal{V})$ is

- a *t-structure* if $\mathcal{U}[1] \subseteq \mathcal{U}$
- *compactly generated* if there is a set $\mathcal{S} \subseteq \mathrm{K}^b(\operatorname{proj} A)$ such that

$$\mathcal{V} = \operatorname{Ker} \operatorname{Hom}_{D(A)}(\mathcal{S}, -).$$

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Every silting complex σ in $\mathrm{K}^b(\operatorname{Proj} A)$ gives rise to a compactly generated t-structure $(\sigma^{\perp_{>0}}, \sigma^{\perp_{\leq 0}})$ in $D(A)$.

Hereditary algebras: a construction

Let A be a finite dimensional hereditary algebra, and let

$$\cdots \leq \lambda_{n-1} \leq \lambda_n \leq \lambda_{n+1} \leq \cdots$$

be a chain of homological ring epimorphisms $\lambda_n : A \to B_n$,

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$\mathcal{D}_n$ the corresponding minimal silting classes.

Then
there is a compactly generated t-structure $(\mathcal{U}, \mathcal{V})$ in $D(A)$ with

$$\mathcal{U} = \{X \in D(A) \mid H^{-n}(X) \in \mathcal{D}_n \cap \mathcal{X}_{n+1} \text{ for all } n \in \mathbb{Z}\}.$$

Hereditary algebras: a construction

Finite chains

$$0_A \leq \lambda_1 \leq \cdots \leq \lambda_m \leq \mathrm{id}_A$$

give rise to t-structures of the form

$$(\sigma^{\perp_{>0}}, \sigma^{\perp_{\leq 0}})$$

for a silting complex σ in $\mathrm{K}^b(\mathrm{Proj}\,A)$.

HEREDITARY ALGEBRAS: A CONSTRUCTION

The silting complex is

$$\sigma = \bigoplus \mathrm{Cone}(\mu_n)[n]$$

where

$$\begin{array}{ccc} A & \xrightarrow{\lambda_{n+1}} & B_{n+1} \\ & {\scriptstyle \lambda_n} \searrow \quad \swarrow {\scriptstyle \mu_n} & \\ & B_n & \end{array}$$

Hereditary algebras: compact silting

Theorem (A-Hrbek). Let A be a finite dimensional hereditary algebra. The compact silting complexes in $\mathrm{K}^b(\operatorname{proj} A)$ correspond bijectively to finite chains

$$0_A \leq \lambda_1 \leq \cdots \leq \lambda_m \leq \mathrm{id}_A$$

of finite-dimensional homological ring epimorphisms.

The Kronecker algebra

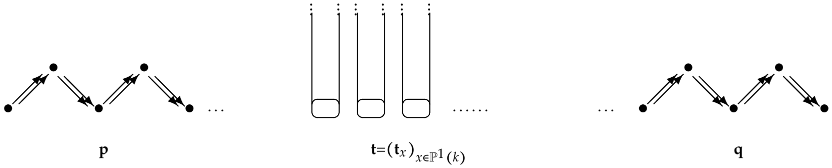
$$A = kQ$$

path algebra of the quiver

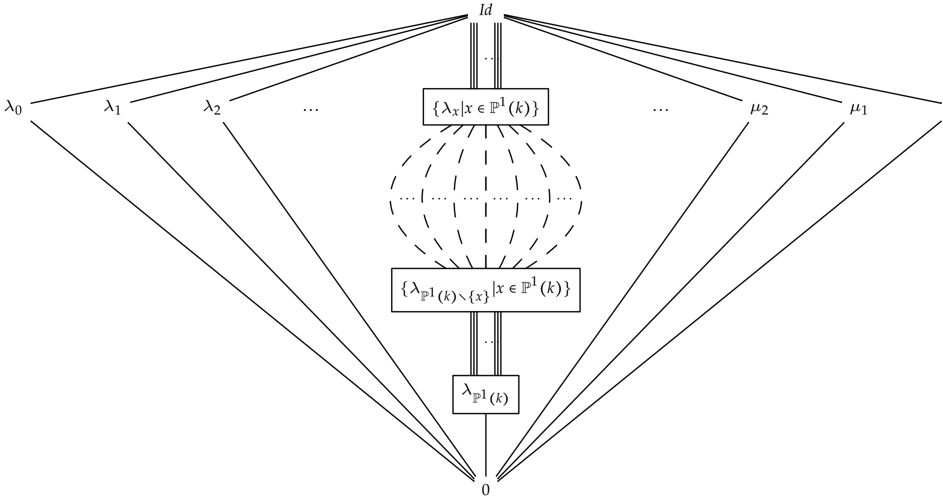
$$Q : \bullet \underset{\beta}{\overset{\alpha}{\rightrightarrows}} \bullet$$

over a field $k = \overline{k}$.

THE KRONECKER ALGEBRA: Auslander-Reiten quiver



THE KRONECKER ALGEBRA: homological ring epimorphisms



The Kronecker algebra: t-structures

Theorem (A-Hrbek). Every compactly generated t-structure $(\mathcal{U}, \mathcal{V})$ in $D(A)$

- corresponds to a chain of homological ring epimorphisms

$$\ldots \leq \lambda_n \leq \lambda_{n+1} \leq \ldots$$

or

- *Happel-Reiten-Smalø-tilt* of the torsion pair $(\mathrm{Add}\,\mathbf{q}, \mathrm{Ker}\mathrm{Hom}_A(\mathbf{q}, -))$, up to shift.

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  where $\mathbb{P}^1(k) \supseteq U_0 \supseteq U_1 \supseteq \cdots \supseteq U_m \supseteq 0$,
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  $\lambda_n : A \to B_n$ = universal localization at modules in U_n;
- the tilting module L with tilting class $\mathcal{D} = \operatorname{KerHom}_A(-, \mathbf{p})$.