

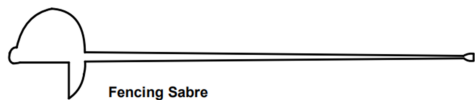
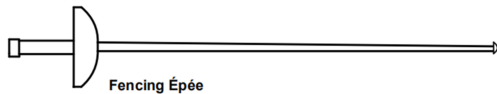
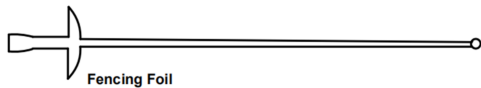
Modeling Player Strength in Fencing

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Introduction to fencing

- Fencing is a combat sport that features sword fighting. It features three weapons, the foil, the épée, and the saber.
- Modern competitive fencing is governed by the Fédération Internationale d'Escrime (FIE), headquartered in Lausanne, Switzerland.
- The national governing body for fencing in the US is the United States Fencing Association (USFA).

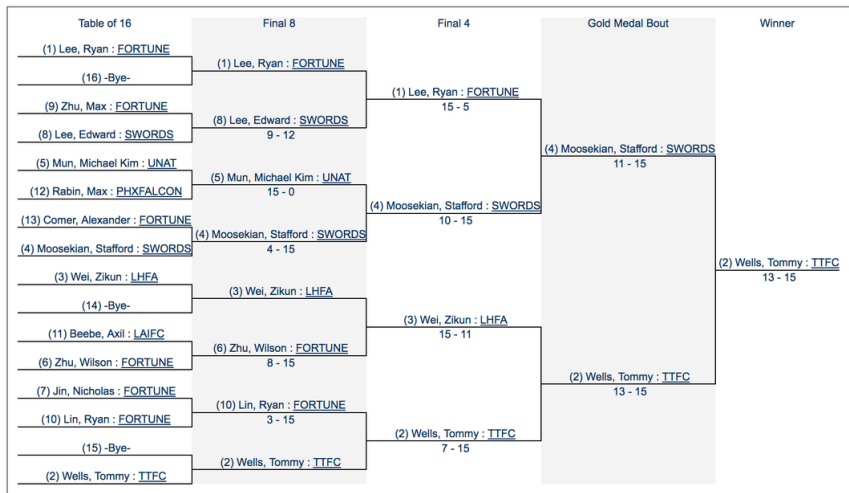
Fencing weapons



Composition of a USA fencing tournament

- A fencing tournament in the USA is usually composed of two phases, the pool phase, and the direct elimination (DE) phase.
- In the pool phase, fencers are separated into groups of size typically 6 to 7. Every fencer fights each other exactly once in their group. The competitors are then ranked according to the points earned, and the last 20% are eliminated.
- The DE phase follows the usual rules of single-elimination/knockout. Players are seeded by their ranks in the pool phase.
- A match between two fencers is called a **bout**. In the pool phase, fencers fence to 5 touches/points, and in the DE phase, 15.

Composition of a USA fencing tournament - visualized



Motivation to the strength modeling

- Both FIE and USFA offer a measure of player strength. However, they are nothing more than the aggregated points that a player has earned in the past year, which is not very accurate.

Data description

- Bout history of the top 50 US fencers in current point standings in each of the six categories (gender/weapon) was scraped from *fencingtracker.com*.
- Data include the scores and fencers of each match, and the ages of the fencers. The bout scores will be analyzed regardless of which stage of tournament they were in.

The Bradley-Terry model

- The Bradley-Terry model is a probability model for the outcome of pairwise comparisons between individuals, teams, or objects.
- The model assumes that the probability of player i beating player j is of the form

$$p_{ij} = P(\text{Player } i \text{ beats player } j) = \frac{\pi_i}{\pi_i + \pi_j},$$

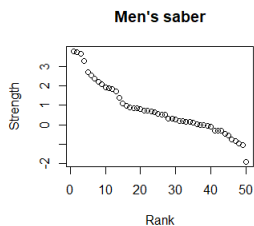
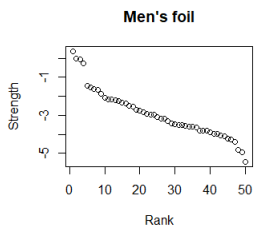
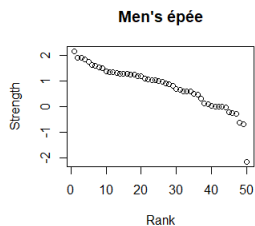
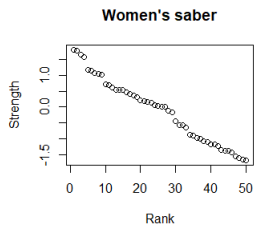
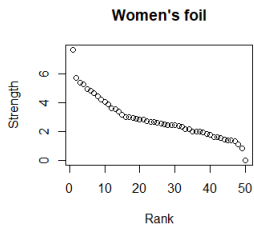
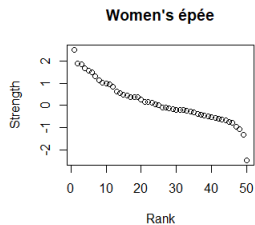
where π_i, π_j are unique quantities associated with player i and j , which can be interpreted as player strength.

- The above equation can also be written as

$$\log\left(\frac{p_{ij}}{1 - p_{ij}}\right) = \log(\pi_i) - \log(\pi_j).$$

I will use $\phi_i = \log(\pi_i)$ instead of π_i as the measure of strength in the following analysis.

Results of the Bradley-Terry model fitting



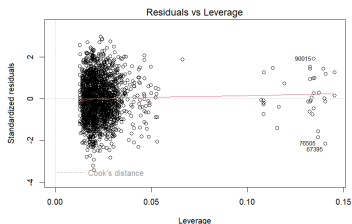
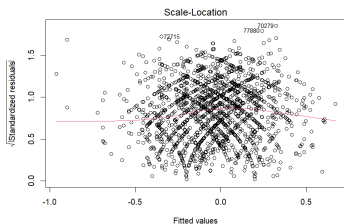
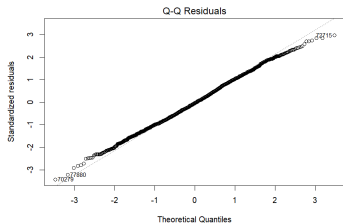
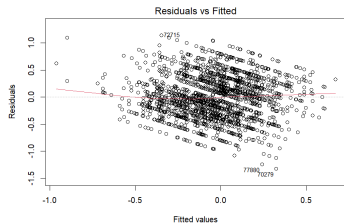
A linear model for modeling the score

- The data also contain the final score for each bout. It may improve on the estimation of the strength if these scores are also included in the regression.
- Suppose the scores in the k -th bout between player i and j are $s_{ijk1} : s_{ijk2}$. Then they can be modeled linearly as

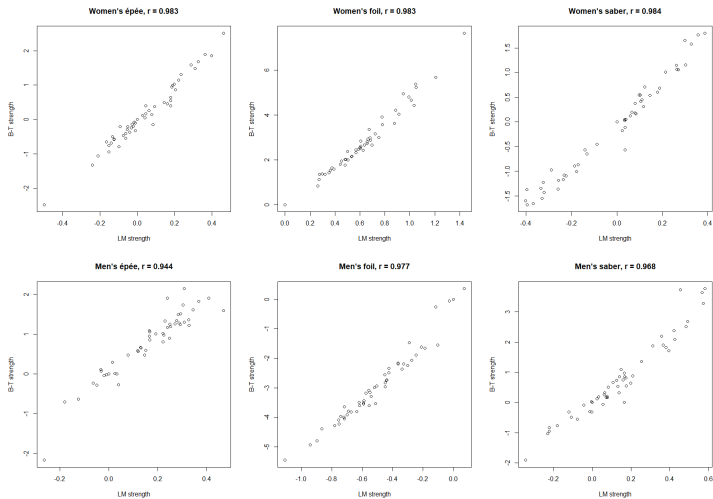
$$\frac{s_{ijk1} - s_{ijk2}}{\max(s_{ijk1}, s_{ijk2})} = \beta_i - \beta_j + e_{ijk},$$

where β_i and β_j are measures of strength, e_{ijk} 's are iid $N(0, \sigma^2)$ random variables.

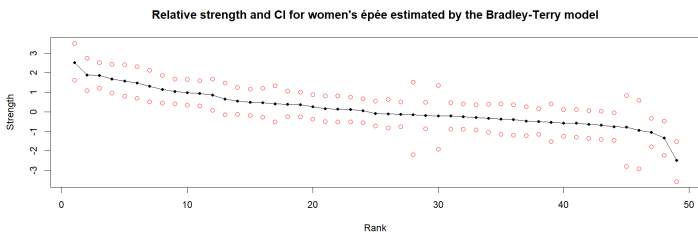
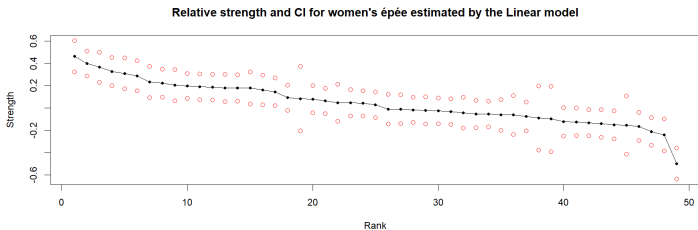
Diagnostics for the linear model applied to women's épée data



Correlation plots of player strength in the two models



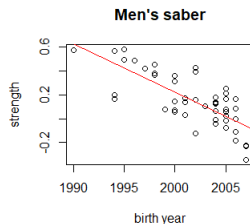
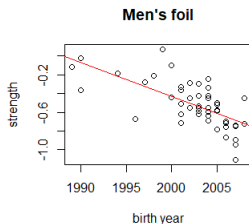
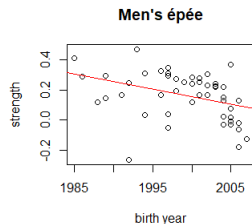
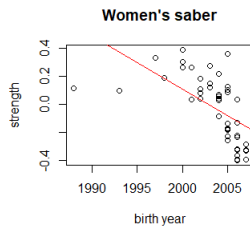
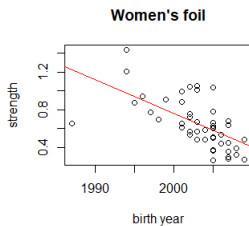
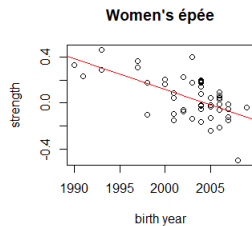
Confidence intervals of player strength in the two models applied to women's épée data



Comparisons of the two models

- The two models perform very similarly in the fencing dataset. The linear model is simpler to compute, easier for interpretation and analysis, and allows for the prediction of match scores. I consider it to be better in this case.
- However, if the bout scores are not accessible, then the Bradley-Terry model should be used.

The relationship between player strength and age



The relationship between player strength and age - analysis

- It appears that the strength is positively correlated with age. Linear effects are tested and turn out to be very significant in all six categories.
- One possible explanation is that my data selection is biased. Only the fencers with very high points are selected. In these players, because the younger ones are naturally more active in participating tournaments, they have higher points than what they should have for their strength. Therefore, weaker players are more likely to be selected if they are younger.