Chapter 5

5.1 a) $\underset{\sim}{\bar{x}} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$; $S = \begin{bmatrix} 8 & -10/3 \\ -10/3 & 2 \end{bmatrix}$

$T^2 = 150/11 = 13.64$

b) T^2 is $3F_{2,2}$ (see (5-5))

c) $H_0: \underset{\sim}{\mu}' = [7,11]$

$\alpha = .05$ so $F_{2,2}(.05) = 19.00$

Since $T^2 = 13.64 < 3F_{2,2}(.05) = 3(19) = 57$; do not reject H_0 at the $\alpha = .05$ level

5.3 a) $$T^2 = \frac{(n-1)\left|\sum_{j=1}^{n}(\underset{\sim}{x}_j-\underset{\sim}{\mu}_0)(\underset{\sim}{x}_j-\underset{\sim}{\mu}_0)'\right|}{\left|\sum_{j=1}^{n}(\underset{\sim}{x}_j-\underset{\sim}{\bar{x}})(\underset{\sim}{x}_j-\underset{\sim}{\bar{x}})'\right|} - (n-1) = \frac{3(244)}{44} - 3 = 13.64$$

b) $$\Lambda = \left(\frac{\left|\sum_{j=1}^{n}(\underset{\sim}{x}_j-\underset{\sim}{\bar{x}})(\underset{\sim}{x}_j-\underset{\sim}{\bar{x}})'\right|}{\left|\sum_{j=1}^{n}(\underset{\sim}{x}_j-\underset{\sim}{\mu}_0)(\underset{\sim}{x}_j-\underset{\sim}{\mu}_0)'\right|}\right)^{n/2} = \left(\frac{44}{244}\right)^2 = .0325$$

Wilks' lambda = $\Lambda^{2/n} = \Lambda^{1/2} = \sqrt{.0325} = .1803$

5.5 $H_0: \underset{\sim}{\mu}' = [.55,.60]$; $T^2 = 1.17$

$\alpha = .05$; $F_{2,40}(.05) = 3.23$

Since $T^2 = 1.17 < \frac{2(41)}{40} F_{2,40}(.05) = 2.05(3.23) = 6.62$,

we do not reject H_0 at the $\alpha = .05$ level. The result is consistent with the 95% confidence ellipse for $\underset{\sim}{\mu}$ pictured in Figure 5.1 since $\underset{\sim}{\mu}' = [.55,.60]$ is <u>inside</u> the ellipse.

5.8 $$\underset{\sim}{a} = S^{-1}(\bar{\underset{\sim}{x}}-\underset{\sim}{\mu}_0) = \begin{bmatrix} 227.273 & -181.818 \\ -181.818 & 212.121 \end{bmatrix} \left(\begin{bmatrix} .564 \\ .603 \end{bmatrix} - \begin{bmatrix} .55 \\ .60 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2.636 \\ -1.909 \end{bmatrix}$$

$$t^2 = \frac{n(\underset{\sim}{a}'(\bar{\underset{\sim}{x}}-\underset{\sim}{\mu}_0))^2}{\underset{\sim}{a}' S \underset{\sim}{a}} = \frac{42\left([2.636 \ -1.909] \begin{bmatrix} .014 \\ .003 \end{bmatrix}\right)^2}{[2.636 \ -1.909] \begin{bmatrix} .0144 & .0117 \\ .0117 & .0146 \end{bmatrix} \begin{bmatrix} 2.636 \\ -1.909 \end{bmatrix}} = 1.31 = T^2$$

5.9 a) Large sample 95% T^2 simultaneous confidence intervals:

Weight:	(69.56, 121.48)	Girth:	(83.49, 103.29)
Body length:	(152.17, 176.59)	Head length:	(16.55, 19.41)
Neck:	(49.61, 61.77)	Head width:	(29.04, 33.22)

b) 95% confidence region determined by all μ_1, μ_4 such that

$$(95.52-\mu_1, 93.39-\mu_4)\begin{bmatrix} .002799 & -.006927 \\ -.006927 & .019248 \end{bmatrix}\begin{pmatrix} 95.52-\mu_1 \\ 93.39-\mu_4 \end{pmatrix} \le 12.59/61 = .2064$$

Beginning at the center $\underset{\sim}{\bar{x}}' = (95.52, 93.39)$, the axes of the 95% confidence ellipsoid are:

major axis $\pm\sqrt{3695.52}\sqrt{12.59}\begin{pmatrix} .939 \\ .343 \end{pmatrix}$

minor axis $\pm\sqrt{45.92}\sqrt{12.59}\begin{pmatrix} -.343 \\ .939 \end{pmatrix}$

(See confidence ellipsoid in part d.)

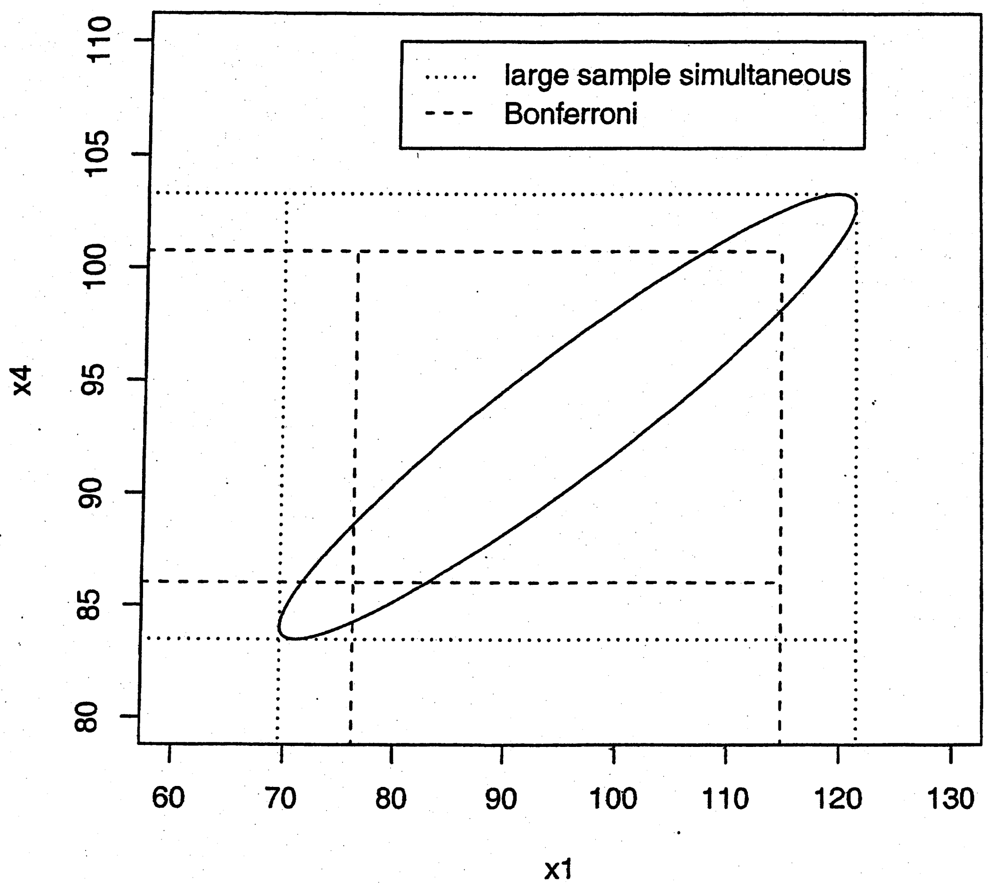
c) Bonferroni 95% simultaneous confidence intervals ($m = 6$):
$t_{60}(.025/6) = 2.728$ (Alternative multiplier is $z(.025/6) = 2.638$)

Weight:	(75.56, 115.48)	Girth:	(86.27, 100.51)
Body length:	(155.00, 173.76)	Head length:	(16.88, 19.08)
Neck:	(51.01, 60.37)	Head width:	(29.52, 32.74)

d) Because of the high positive correlation between weight (X_1) and girth (X_4), the 95% confidence ellipse is smaller, more informative, than the 95% Bonferroni rectangle.

5.9 (Continued)

Large sample 95% confidence regions.



e) Bonferroni 95% simultaneous confidence interval for difference between mean head width and mean head length ($\mu_6 - \mu_5$) follows.
($m = 7$ to allow for new statement and statements about individual means):
$t_{60}(.025/7) = 2.783$ (Alternative multiplier is $z(.025/7) = 2.690$)

$$\bar{x}_6 - \bar{x}_5 \pm t_{60}(.0036)\sqrt{\frac{s_{66} - 2s_{56} + s_{55}}{n}} = (31.13 - 17.98) \pm 2.783\sqrt{\frac{21.26 - 2(13.88) + 9.95}{61}}$$

or $$12.49 \le \mu_6 - \mu_5 \le 13.81$$

5.10 **a)** 95% T^2 simultaneous confidence intervals:

Lngth2: (130.65, 155.93) Lngth4: (160.33, 185.95)
Lngth3: (127.00, 191.58) Lngth5: (155.37, 198.91)

b) 95% T^2 simultaneous intervals for change in length (ΔLngth):

ΔLngth2-3: (–21.24, 53.24)
ΔLngth3-4: (–22.70, 50.42)
ΔLngth4-5: (–20.69, 28.69)

c) 95% confidence region determined by all μ_{2-3}, μ_{4-5} such that

$$(16-\mu_{2-3}, 4-\mu_{4-5})\begin{bmatrix} .011024 & .009386 \\ .009386 & .025135 \end{bmatrix}\begin{pmatrix} 16-\mu_{2-3} \\ 4-\mu_{4-5} \end{pmatrix} \le 72.96/7 = 10.42$$

where μ_{2-3} is the mean increase in length from year 2 to 3, and μ_{4-5} is the mean increase in length from year 4 to 5.

Beginning at the center $\underset{\sim}{\bar{x}}' = (16,4)$, the axes of the 95% confidence ellipsoid are:

$$\text{major axis} \qquad \pm\sqrt{157.8}\sqrt{72.96}\begin{pmatrix} .895 \\ -.447 \end{pmatrix}$$

$$\text{minor axis} \qquad \pm\sqrt{33.53}\sqrt{72.96}\begin{pmatrix} .447 \\ .895 \end{pmatrix}$$

(See confidence ellipsoid in part e.)

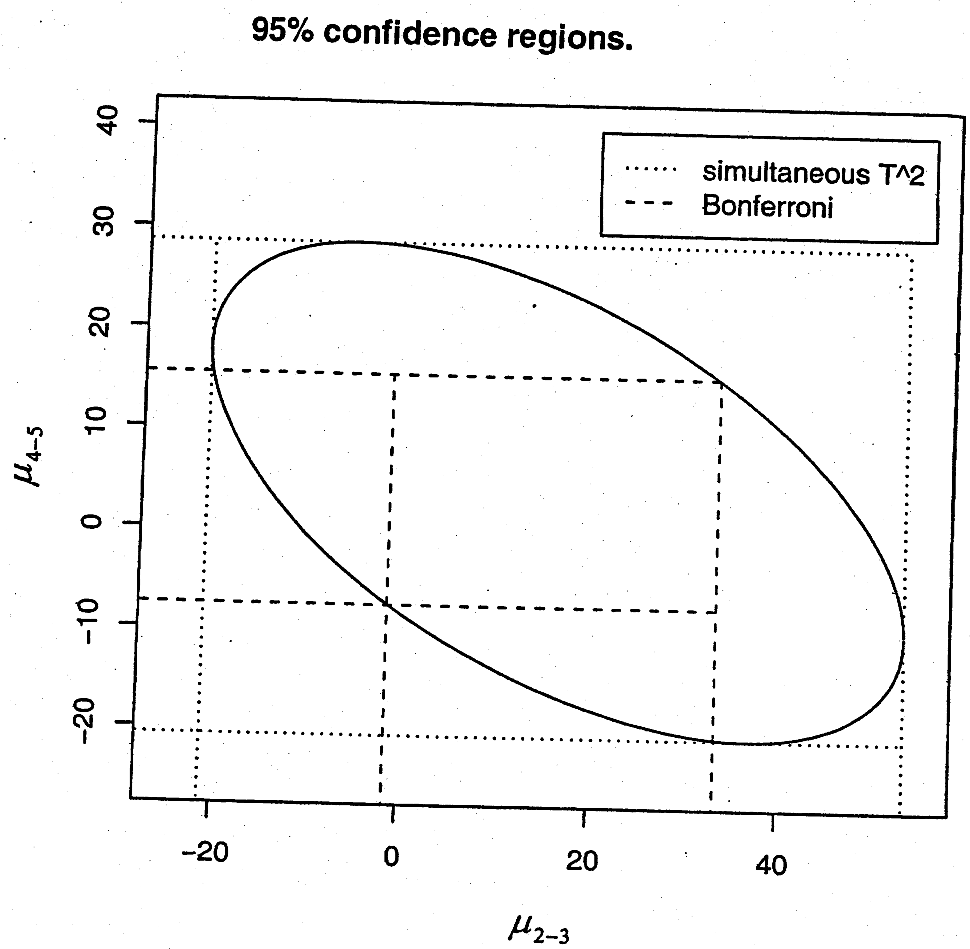
d) Bonferroni 95% simultaneous confidence intervals ($m = 7$):

Lngth2: (137.37, 149.21) Lngth4: (167.14, 179.14)
Lngth3: (144.18, 174.40) Lngth5: (166.95, 187.33)

ΔLngth2-3: (–1.43, 33.43) ΔLngth4-5: (–7.55, 15.55)
ΔLngth3-4: (–3.25, 30.97)

5.10 (Continued)

e) The Bonferroni 95% confidence rectangle is much smaller and more informative than the 95% confidence ellipse.



5.11 a) $\bar{\underset{\sim}{x}}' = [5.1856,\ 16.0700]$

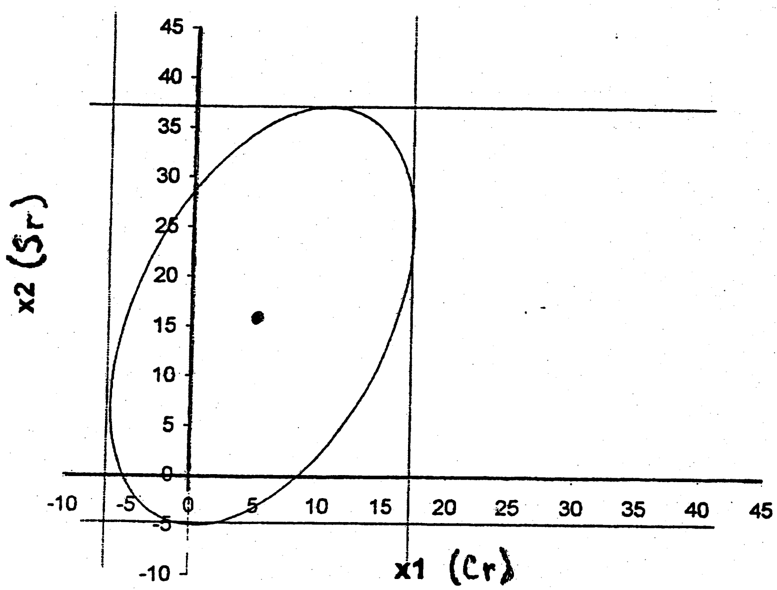
$$S = \begin{bmatrix} 176.0042 & 287.2412 \\ 287.2412 & 527.8493 \end{bmatrix}; \quad S^{-1} = \begin{bmatrix} .0508 & -.0276 \\ -.0276 & .0169 \end{bmatrix}$$

Eigenvalues and eigenvectors of S:

$$\hat{\lambda}_1 = 688.759 \qquad \hat{\underset{\sim}{e}}_1' = (.49, .87)$$
$$\hat{\lambda}_2 = 15.094 \qquad \hat{\underset{\sim}{e}}_2' = (.87, -.49)$$

$$\frac{(n-1)p}{(n-p)} F_{p,n-p}(.10) = \frac{8(2)}{7} F_{2,7}(.10) = \frac{16}{7}(3.26) = 7.45$$

Confidence Region



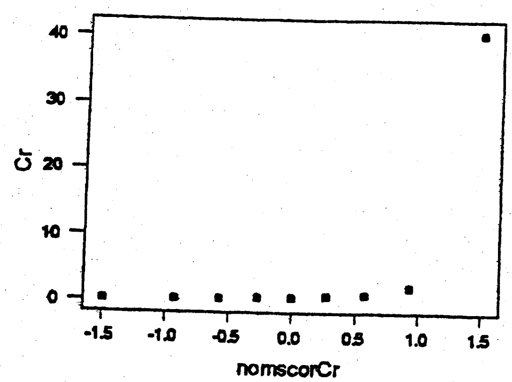
b) 90% T^2 intervals for the full data set:

Cr: (-6.88, 17.25) Sr: (-4.83, 36.97)

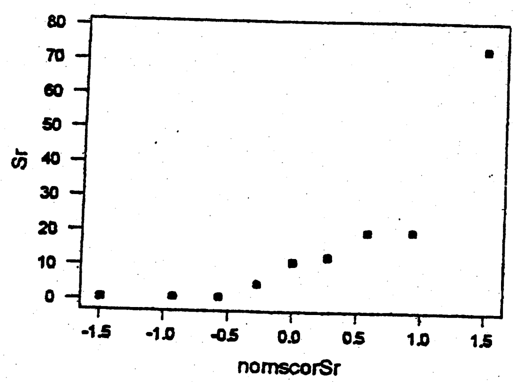
$[.30, 10]'$ is a plausible value for $\underset{\sim}{\mu}$

5.11 (Continued)

c) Q-Q plots for the marginal distributions of both variables



Since $r = 0.627$ we reject the hypothesis of normality for this variable at $\alpha = 0.01$



Since $r = 0.818$ we reject the hypothesis of normality for this variable at $\alpha = 0.01$

d) With data point (40.53, 73.68) removed,

$$\underset{\sim}{\bar{x}}' = [.7675, 8.8688]; \qquad S = \begin{bmatrix} .3786 & 1.0303 \\ 1.0303 & 69.8598 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 2.7518 & -.0406 \\ -.0406 & .0149 \end{bmatrix}$$

$$\frac{(n-1)p}{(n-p)} F_{p,n-p}(.10) = \frac{7(2)}{6} F_{2,6}(.10) = \frac{14}{6}(3.46) = 8.07$$

90% T^2 intervals: Cr: (.15, 1.39) Sr: (.47, 17.27)

5.12 Initial estimates are

$$\tilde{\mu} = \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \quad \tilde{\Sigma} = \begin{bmatrix} 0.5 & 0.0 & 0.5 \\ & 2.0 & 0.0 \\ & & 1.5 \end{bmatrix}.$$

The first revised estimates are

$$\tilde{\mu} = \begin{bmatrix} 4.0833 \\ 6.0000 \\ 2.2500 \end{bmatrix}, \quad \tilde{\Sigma} = \begin{bmatrix} 0.6042 & 0.1667 & 0.8125 \\ & 2.500 & 0.0 \\ & & 1.9375 \end{bmatrix}.$$

5.13 The χ^2 distribution with 3 degrees of freedom.

5.14 Length of one-at-a time t-interval / Length of Bonferroni interval $= t_{n-1}(\alpha/2)/t_{n-1}(\alpha/2m)$.

	m		
n	2	4	10
15	0.8546	0.7489	0.6449
25	0.8632	0.7644	0.6678
50	0.8691	0.7749	0.6836
100	0.8718	0.7799	0.6910
∞	0.8745	0.7847	0.6983

5.15

(a).

$$\begin{aligned} E(X_{ij}) &= (1)p_i + (0)(1-p_i) = p_i. \\ Var(X_{ij}) &= (1-p_i)^2 p_i + (0-p_i)^2(1-p_i) = p_i(1-p_i) \end{aligned}$$

(b). $Cov(X_{ij}, X_{kj}) = E(X_{ij}X_{ik}) - E(X_{ij})E(X_{kj}) = 0 - p_ip_k = -p_ip_k.$

5.16

(a). Using $\hat{p}_j \pm \sqrt{\chi_4^2(0.05)}\sqrt{\hat{p}_j(1-\hat{p}_j)/n}$, the 95 % confidence intervals for p_1, p_2, p_3, p_4, p_5 are
(0.221, 0.370), (0.258, 0.412), (0.098, 0.217), (0.029, 0.112), (0.084, 0.198) respectively.

(b). Using $\hat{p}_1 - \hat{p}_2 \pm \sqrt{\chi_4^2(0.05)}\sqrt{(\hat{p}_1(1-\hat{p}_1) + \hat{p}_2(1-\hat{p}_2) - 2\hat{p}_1\hat{p}_2)/n}$, the 95 % confidence interval for $p_1 - p_2$ is (−0.118, 0.0394). There is no significant difference in two proportions.

5.17

$\hat{p}_1 = 0.585$, $\hat{p}_2 = 0.310$, $\hat{p}_3 = 0.105$. Using $\hat{p}_j \pm \sqrt{\chi_3^2(0.05)}\sqrt{\hat{p}_j(1-\hat{p}_j)/n}$, the 95 % confidence intervals for p_1, p_2, p_3 are (0.488, 0.682), (0.219, 0.401), (0.044, 0.166), respectively.

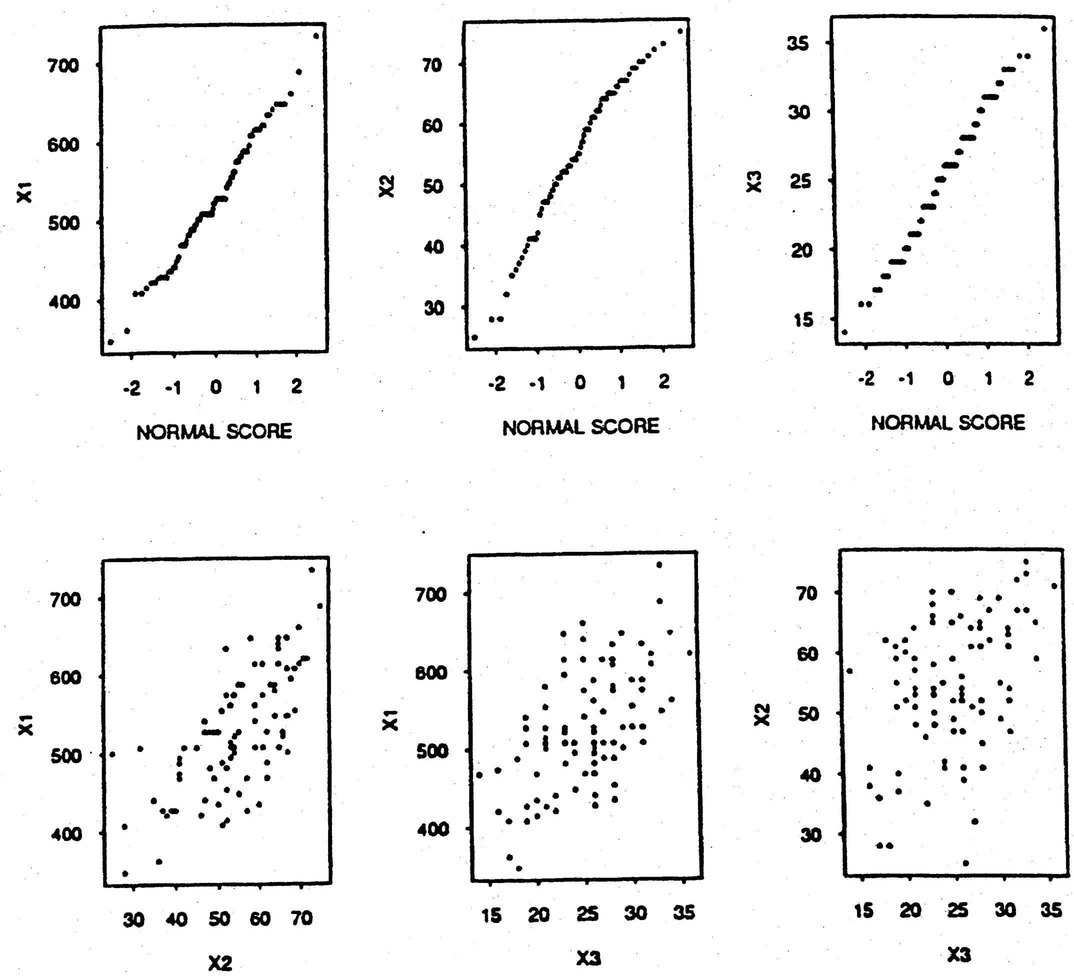
5.18

(a). Hotelling's $T^2 = 223.31$. The critical point for the statistic ($\alpha = 0.05$) is 8.33. We reject $H_0 : \mu = (500, 50, 30)'$. That is, The group of students represented by scores are significantly different from average college students.

(b). The lengths of three axes are 23.730, 2.473, 1.183. And directions of corresponding axes are

$$\begin{pmatrix} 0.994 \\ 0.103 \\ 0.038 \end{pmatrix}, \quad \begin{pmatrix} -0.104 \\ 0.995 \\ 0.006 \end{pmatrix}, \quad \begin{pmatrix} -0.037 \\ -0.010 \\ 0.999 \end{pmatrix}.$$

(c). Data look fairly normal.



5.19 a) The summary statistics are:

$$n = 30, \quad \bar{\underset{\sim}{x}} = \begin{bmatrix} 1860.50 \\ 8354.13 \end{bmatrix} \quad \text{and} \quad S = \begin{bmatrix} 124055.17 & 361621.03 \\ 361621.03 & 3486330.90 \end{bmatrix}$$

where S has eigenvalues and eigenvectors

$$\lambda_1 = 3407292 \qquad \underset{\sim}{e}_1' = [.105740, .994394]$$

$$\lambda_2 = 82748 \qquad \underset{\sim}{e}_2' = [.994394, -.105740]$$

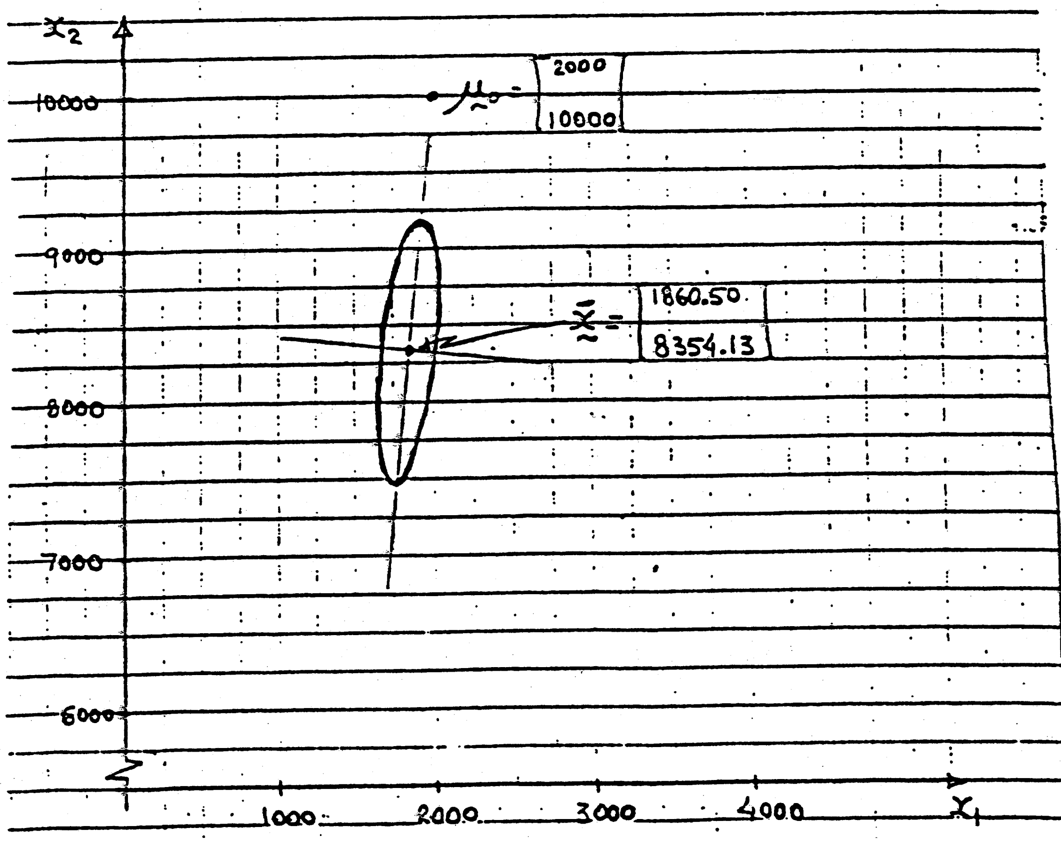
Then, since $\frac{1}{n}\frac{p(n-1)}{n-p}F_{p,n-p}(\alpha) = \frac{1}{30}\;\frac{2(29)}{28}F_{2,28}(.05) = .2306$,

a 95% confidence region for $\underset{\sim}{\mu}$ is given by the set of $\underset{\sim}{\mu}$

$$[1860.50-\mu_1,\ 8354.13-\mu_2]\begin{bmatrix}124055.17 & 361621.03\\ 361621.03 & 3486330.90\end{bmatrix}^{-1}\begin{bmatrix}1860.50-\mu_1\\ 8354.13-\mu_2\end{bmatrix}$$

$$\leq .2306$$

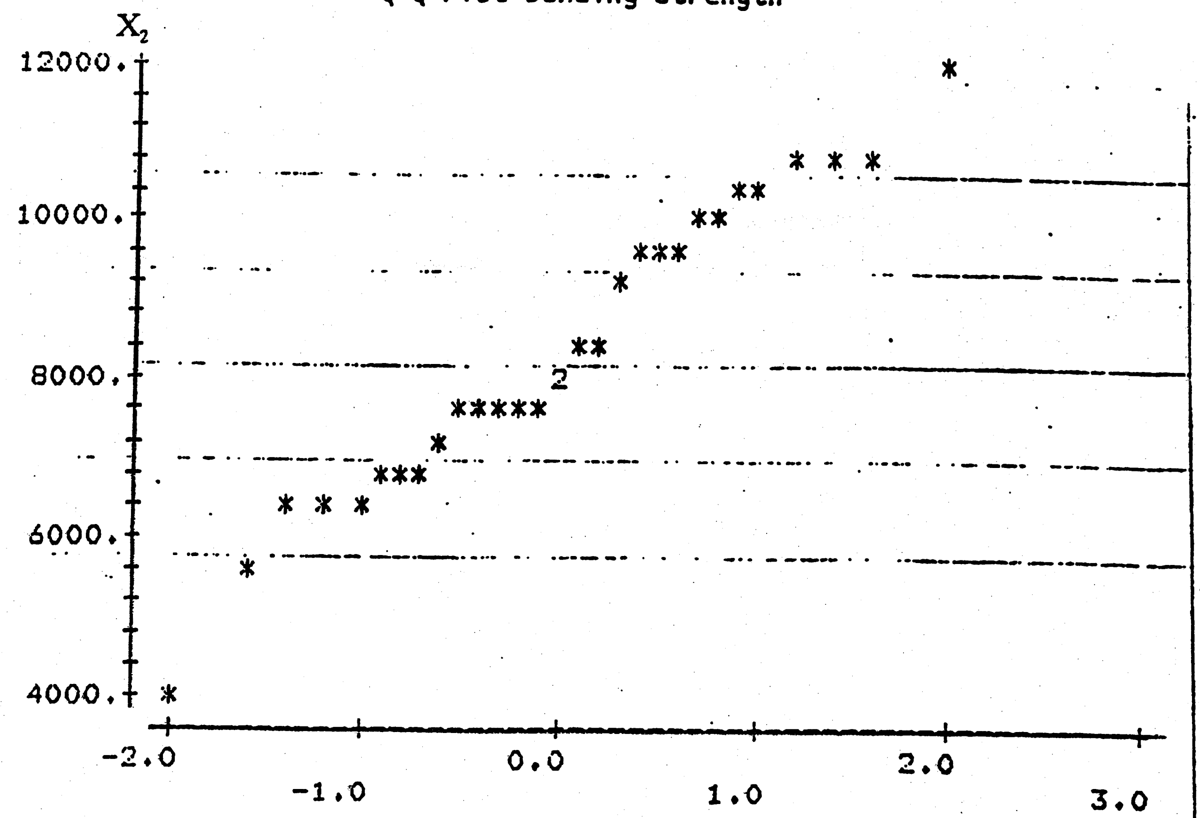
The half lengths of the axes of this ellipse are $\sqrt{.2306}\,\sqrt{\lambda_1} = 886.4$ and $\sqrt{.2306}\,\sqrt{\lambda_2} = 138.1$. Therefore the ellipse has the form



b) Since $\underset{\sim}{\mu}_0 = [2000, 10000]'$ does not fall within the 95% confidence ellipse, we would reject the hypothesis $H_0 : \underset{\sim}{\mu} = \underset{\sim}{\mu}_0$ at the 5% level. Thus, the data analyzed are <u>not</u> consistent with these values.

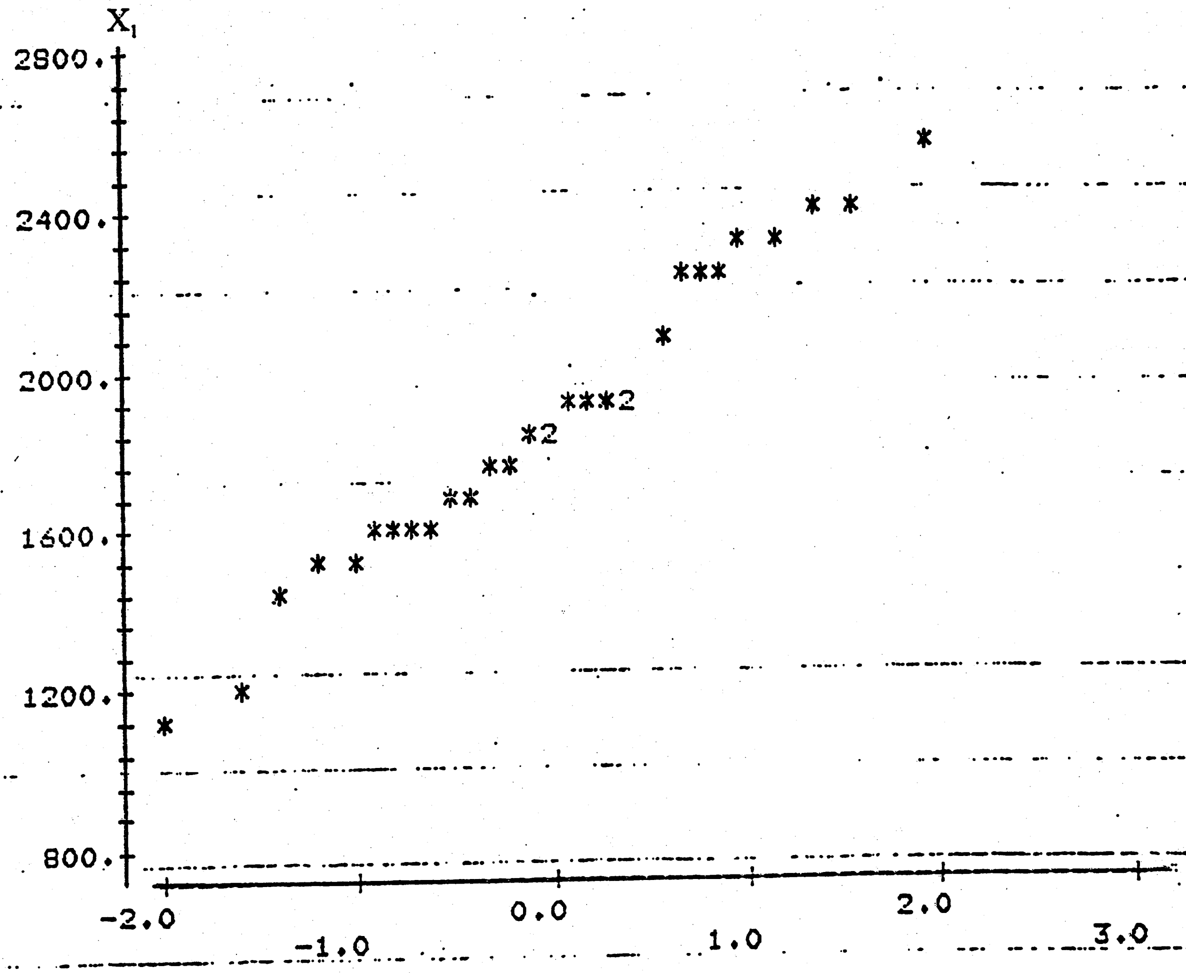
c) The Q-Q plots for both stiffness and bending strength (see below) show that the marginal normality is not seriously violated. Also the correlation coefficients for the test of normality are .989 and .990 respectively so that we fail to reject even at the 1% significance level. Finally, the scatter diagram (see below) does not indicate departure from bivariate normality. So, the bivariate normal distribution is a plausible probability model for these data.

Q-Q Plot-Bending Strength



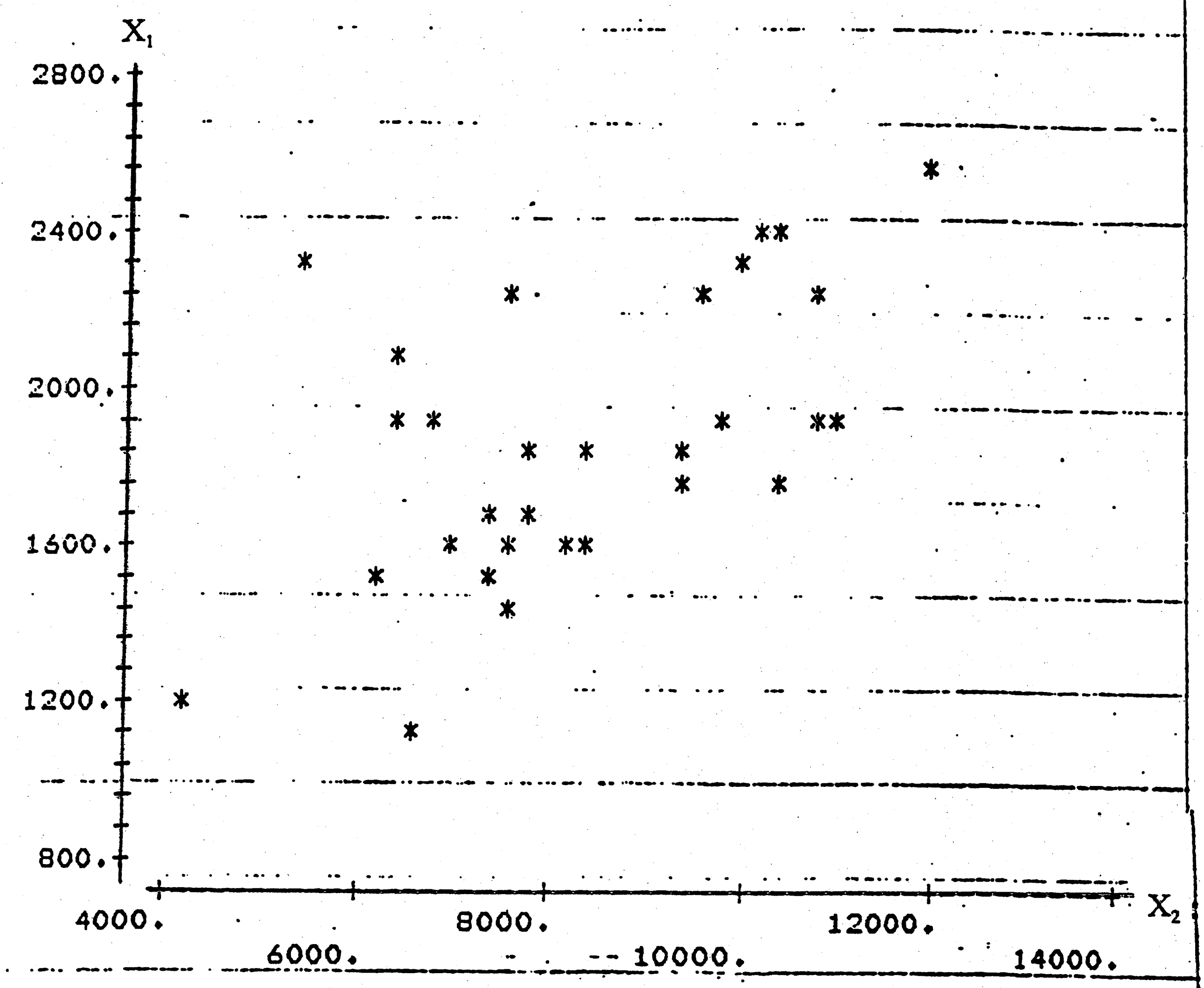
Correlation .989

Q-Q Plot-Stiffness

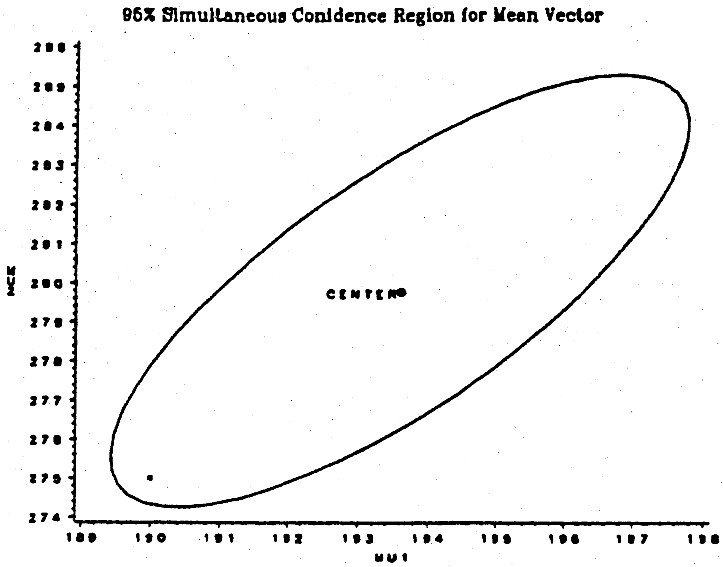


Correlation = .990

Scatter Diagram



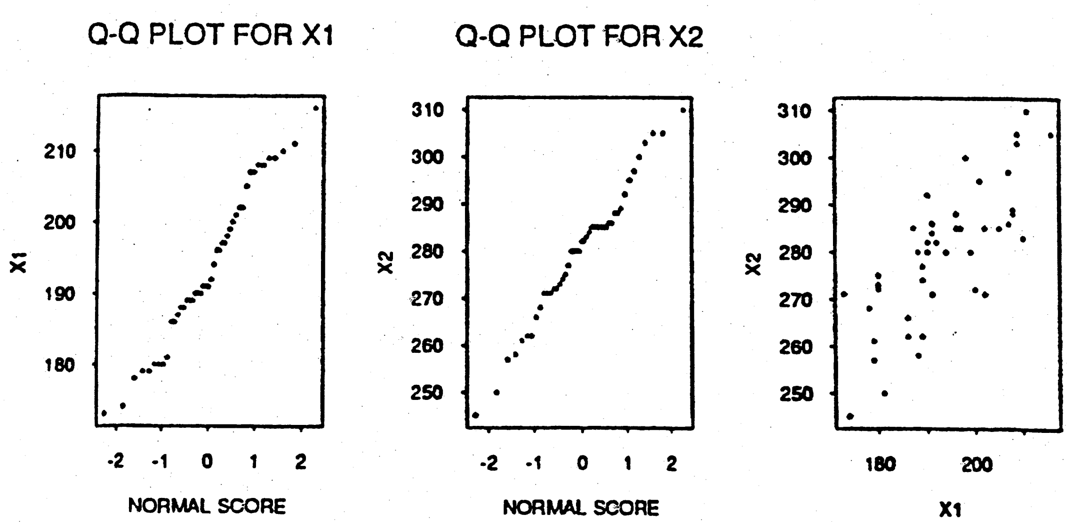
5.20 (a). Yes, they are plausible since the hypothesized vector $\underset{\sim}{\mu}_0$ (denoted as $*$ in the plot) is inside the 95% confidence region.



(b).

	LOWER	UPPER
Bonferroni C. I.:	189.822	197.423
	274.782	284.774
Simultaneous C. I.:	189.422	197.823
	274.256	285.299

Simultaneous confidence intervals are larger than Bonferroni's confidence intervals. Simultaneous confidence intervals will touch the simultaneous confidence region from outside.
(c). Q-Q plots suggests non-normality of (X_1, X_2). Could try transforming X_1.



5.21

HOTELLING T SQUARE = 9.0218
P-VALUE 0.3616

	N	MEAN	STDEV	T2 INTERVAL	TO	BONFERRONI	TO
x1	25	0.84380	0.11402	.742	.946	.778	.909
x2	25	0.81832	0.10685	.723	.914	.757	.880
x3	25	1.79268	0.28347	1.540	2.046	1.629	1.956
x4	25	1.73484	0.26360	1.499	1.970	1.583	1.887
x5	25	0.70440	0.10756	.608	.800	.642	.766
x6	25	0.69384	0.10295	.602	.786	.635	.753

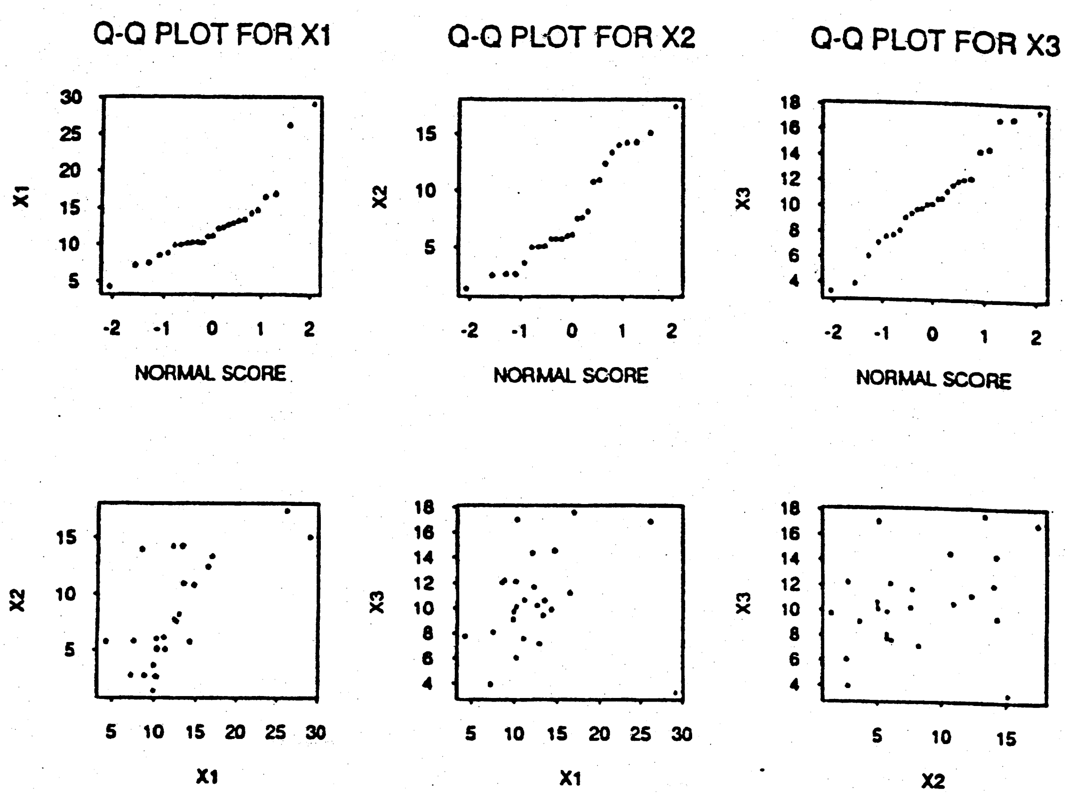
The Bonferroni intervals use t (.00417) = 2.88 and

the T2 intevals use the constant 4.465.

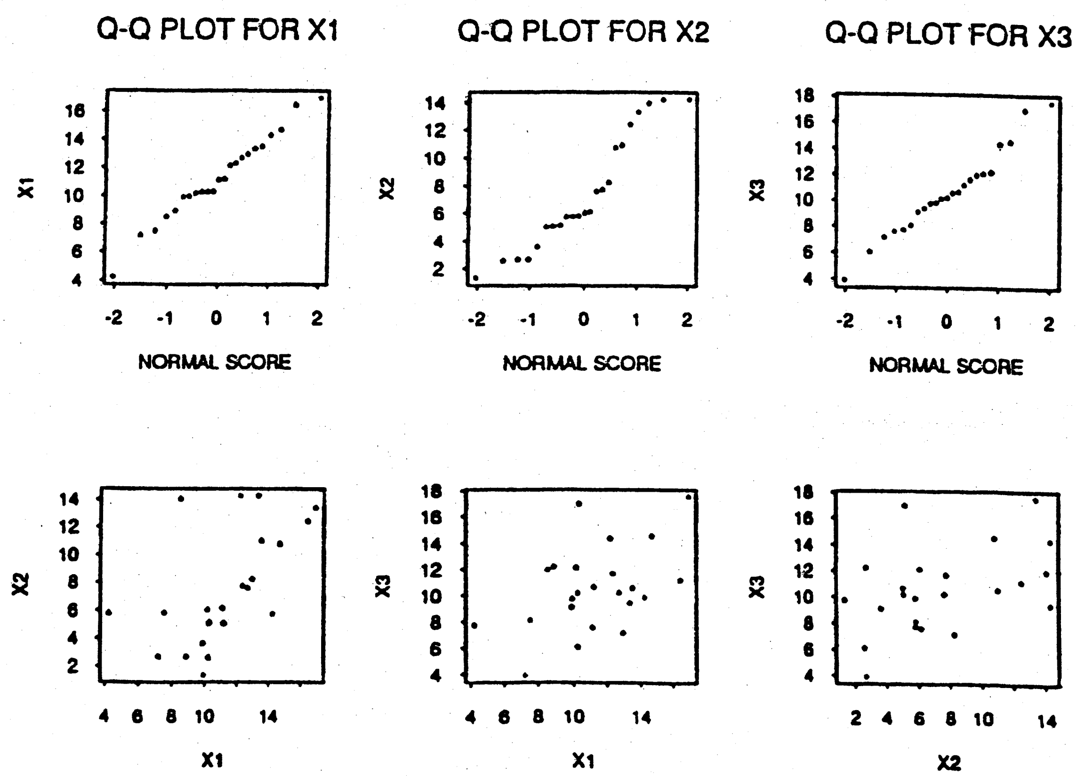
5.22

(a). After eliminating outliers, the approximation to normality is improved.

WITH OUTLIERS



WITHOUT OUTLIERS



(b) Outliers removed:

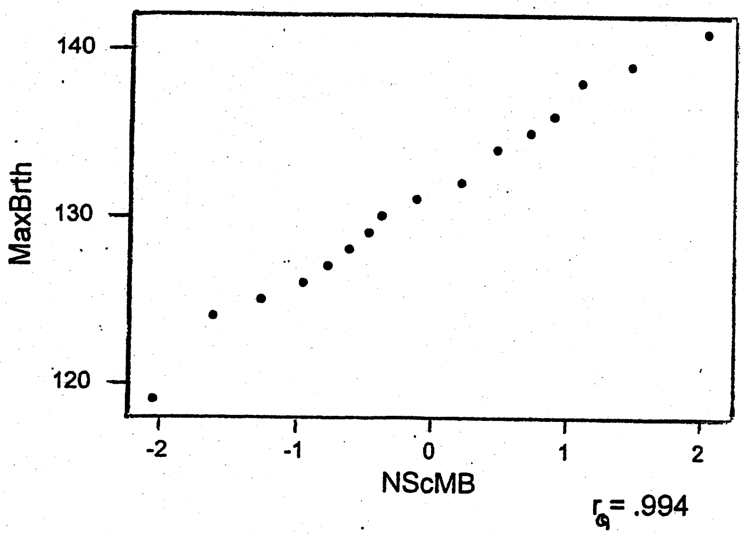
	LOWER	UPPER
Bonferroni C. I.:	9.63	12.87
	5.24	9.67
	8.82	12.34
Simultaneous C. I.:	9.25	13.24
	4.72	10.19
	8.41	12.76

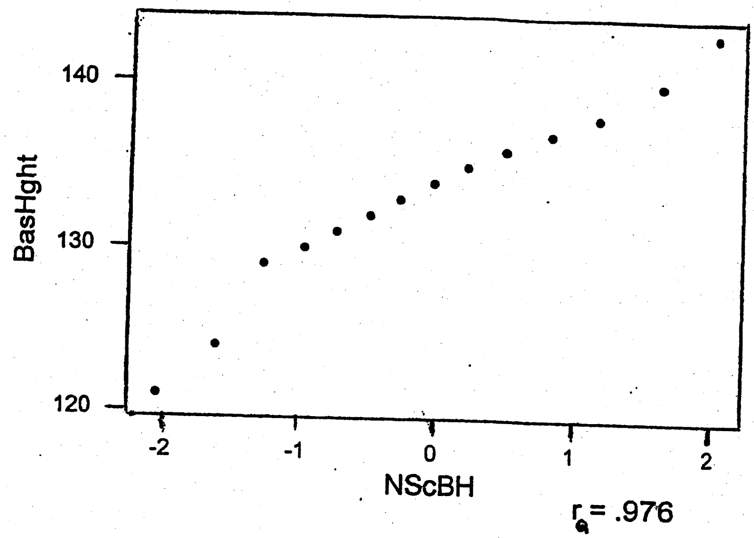
Simultaneous confidence intervals are larger than Bonferroni's confidence intervals.

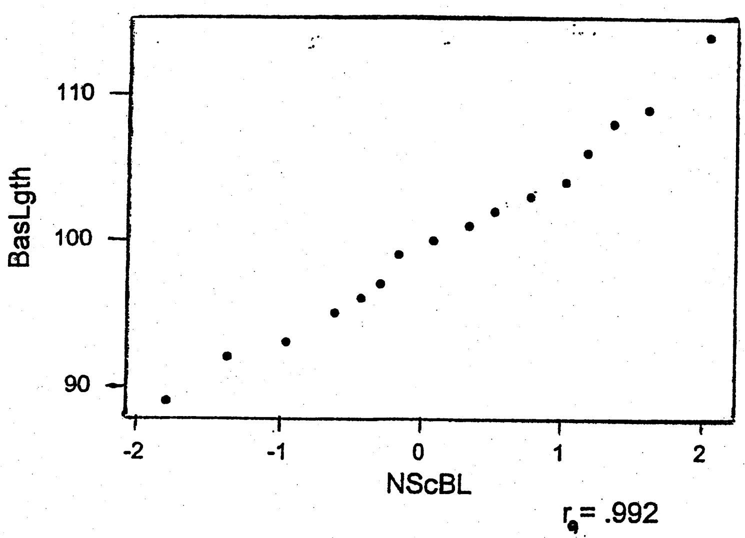
(b) Full data set:

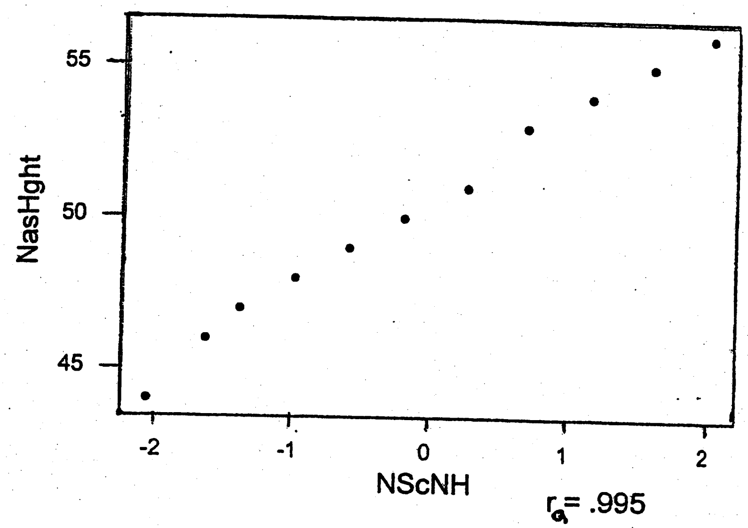
	Lower	Upper
Bonferroni C. I.:	9.79	15.33
	5.78	10.55
	8.65	12.44
Simultaneous C. I.:	9.16	15.96
	5.23	11.09
	8.21	12.87

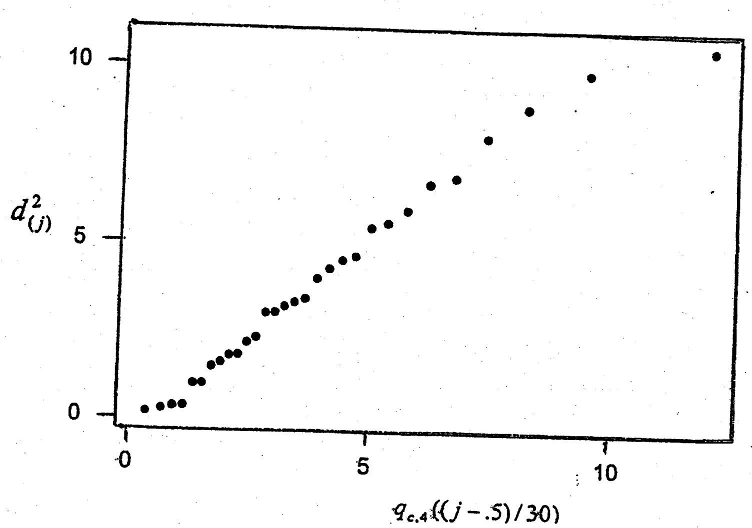
5.23 a) The data appear to be multivariate normal as shown by the "straightness" of the Q-Q plots and chi-square plot below.











5.23 (Continued)

b) Bonferroni 95% simultaneous confidence intervals ($m = p = 4$):
$t_{29}(.05/8) = 2.663$

MaxBrth:	(128.87, 133.87)
BasHgth:	(131.42, 135.78)
BasLngth:	(96.32, 102.02)
NasHgth:	(49.17, 51.89)

95% T^2 simultaneous confidence intervals:

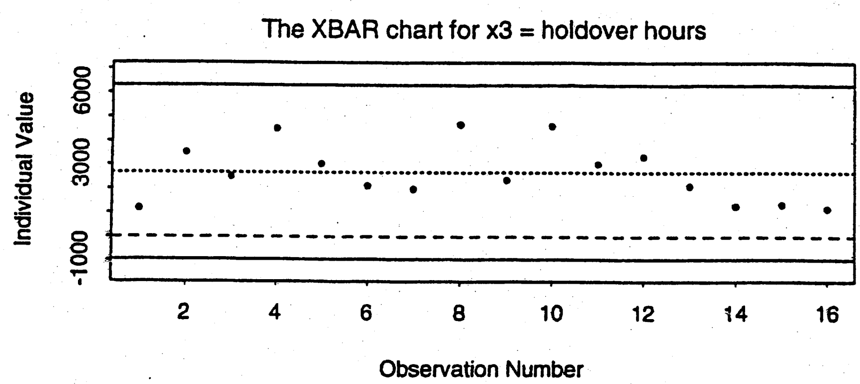
$$\sqrt{\frac{4(29)}{26}F_{4,26}(.05)} = 3.496$$

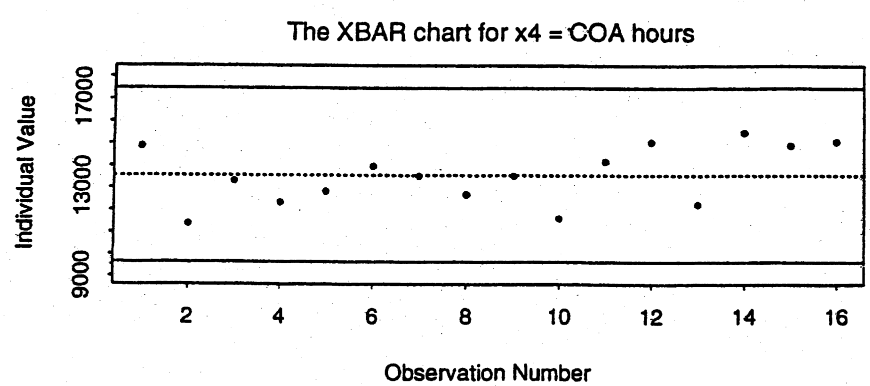
MaxBrth:	(128.08, 134.66)
BasHgth:	(130.73, 136.47)
BasLngth:	(95.43, 102.91)
NasHgth:	(48.75, 52.31)

The Bonferroni intervals are slightly shorter than the T^2 intervals.

5.24 Individual $\overline{X}$ charts for the Madison, Wisconsin, Police Department data

	xbar	s	LCL	UCL	
LegalOT	3557.8	606.5	1738.1	5377.4	
ExtraOT	1478.4	1182.8	-2070.0	5026.9	use LCL = 0
Holdover	2676.9	1207.7	-946.2	6300.0	use LCL = 0
COA	13563.6	1303.2	9654.0	17473.2	
MeetOT	800.0	474.0	-622.1	2222.1	use LCL = 0

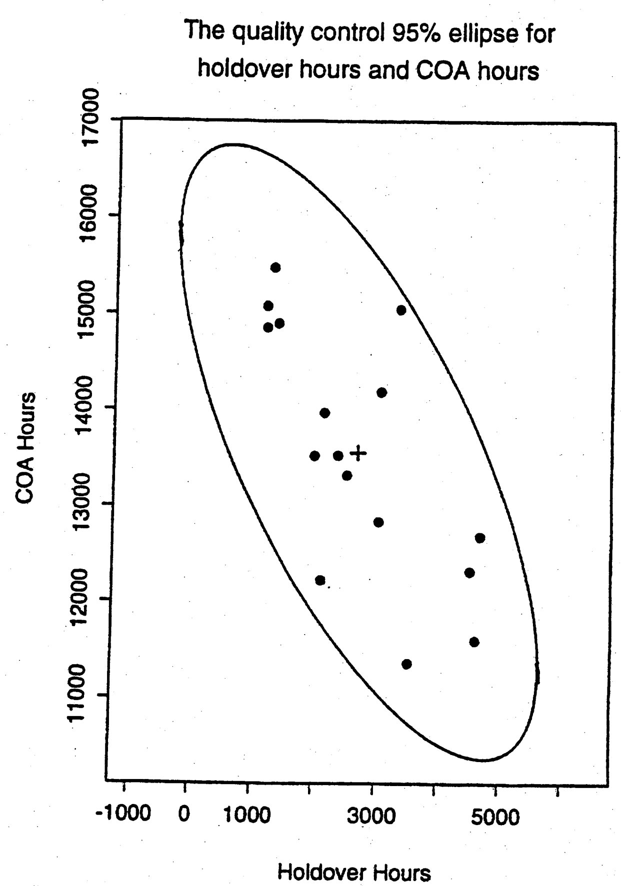


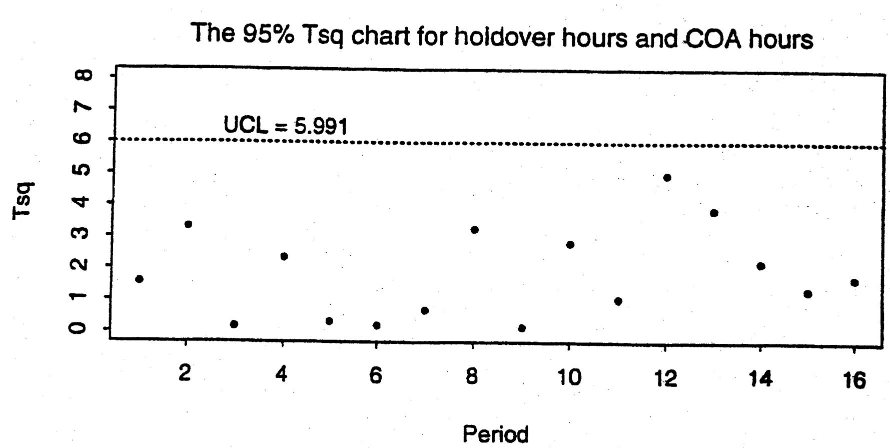


Both holdover and COA hours are stable and in control.

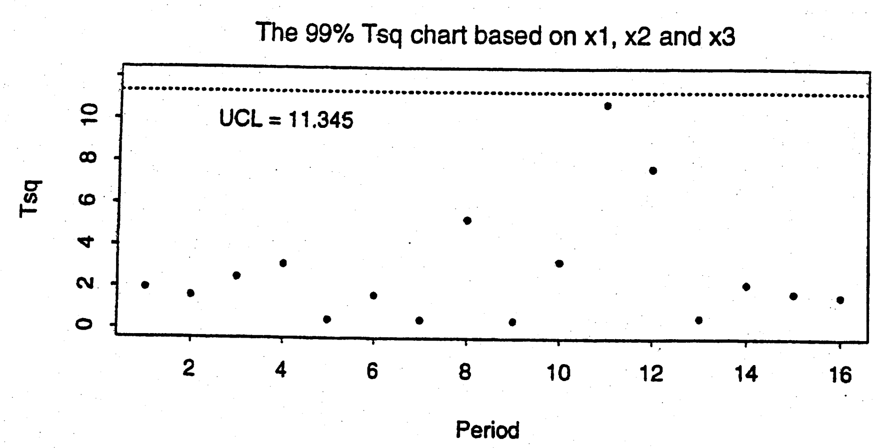
5.25 Quality ellipse and T^2 chart for the holdover and COA overtime hours. All points are in control. The quality control 95% ellipse is

$$1.37\times 10^{-6}(x_3 - 2677)^2 + 1.18\times 10^{-6}(x_4 - 13564)^2$$
$$+1.80\times 10^{-6}(x_3 - 2677)(x_4 - 13564) = 5.99.$$



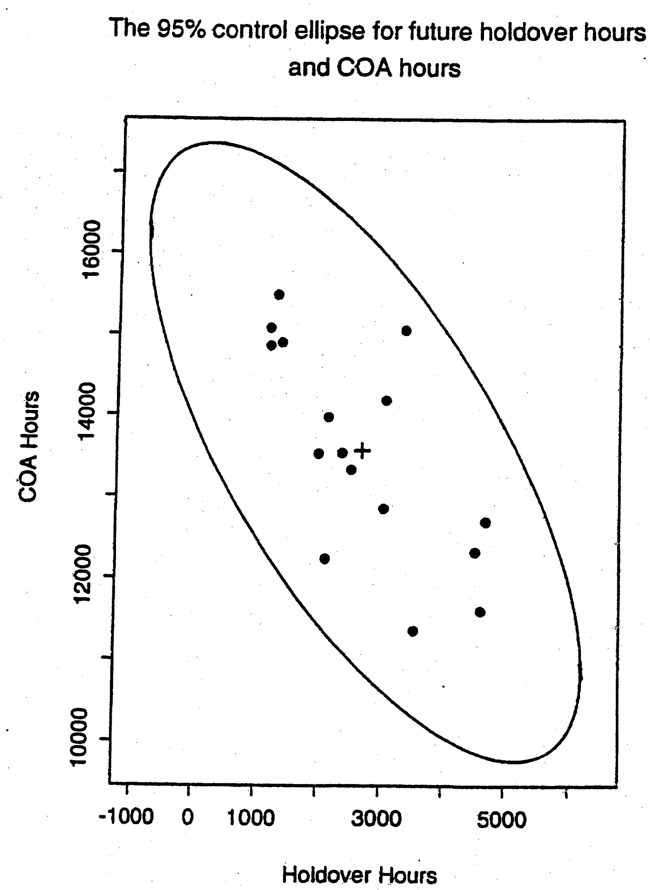


5.26 T^2 chart using the data on x_1 = legal appearances overtime hours, x_2 = extraordinary event overtime hours, and x_3 = holdover overtime hours. All points are in control.



5.27 The 95% prediction ellipse for x_3 = holdover hours and x_4 = COA hours is

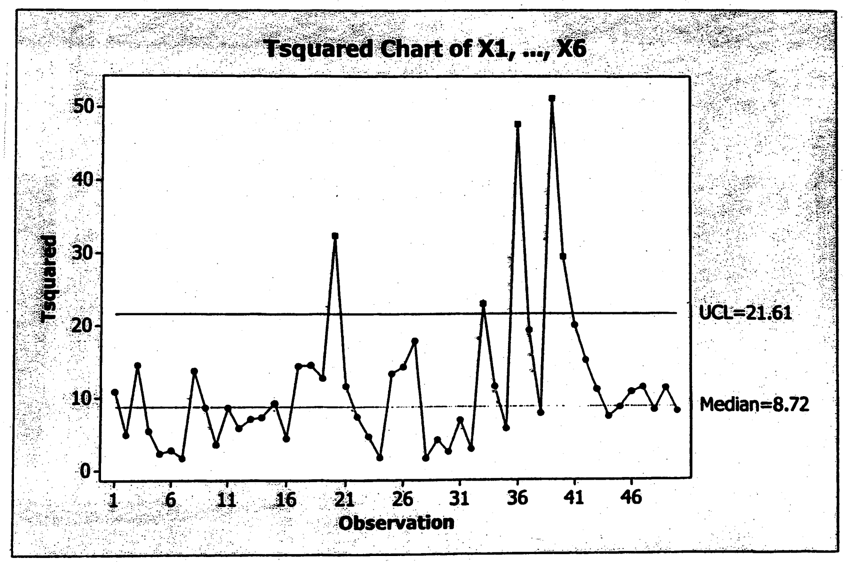
$$1.37\times10^{-6}(x_3-2677)^2+1.18\times10^{-6}(x_4-13564)^2$$
$$+1.80\times10^{-6}(x_3-2677)(x_4-13564)=8.51.$$



5.28 (a)

$$\bar{\mathbf{x}} = \begin{bmatrix} -.506 \\ -.207 \\ -.062 \\ -.032 \\ .698 \\ -.065 \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} .0626 & .0616 & .0474 & .0083 & .0197 & .0031 \\ .0616 & .0924 & .0268 & -.0008 & .0228 & .0155 \\ .0474 & .0268 & .1446 & .0078 & .0211 & -.0049 \\ .0083 & -.0008 & .0078 & .1086 & .0221 & .0066 \\ .0197 & .0228 & .0211 & .0221 & .3428 & .0146 \\ .0031 & .0155 & -.0049 & .0066 & .0146 & .0366 \end{bmatrix}$$

The T^2 chart follows.



(b) Multivariate observations 20, 33, 36, 39 and 40 exceed the upper control limit. The individual variables that contribute significantly to the out of control data points are indicated in the table below.

	Point	Variable	P-Value
Greater Than UCL	20	X1	0.0000
		X2	0.0001
		X3	0.0000
		X4	0.0105
		X5	0.0210
		X6	0.0032
	33	X4	0.0088
		X6	0.0000
	36	X1	0.0000
		X2	0.0000
		X3	0.0000
		X4	0.0343
	39	X2	0.0198
		X4	0.0001
		X5	0.0054
		X6	0.0000
	40	X1	0.0000
		X2	0.0088
		X3	0.0114
		X4	0.0013

5.29 $T^2 = 12.472$. Since $T^2 = 12.472 < \frac{29(6)}{24} F_{6,24}(.05) = 7.25(2.51) = 18.2$, we do not reject $H_0 : \boldsymbol{\mu} = \mathbf{0}$ at the 5% level.

5.30 **(a)** Large sample 95% Bonferroni intervals for the indicated means follow. Multiplier is $t_{49}(.05/2(6)) \approx z(.0042) = 2.635$

Petroleum: $.766 \pm 2.635(.925/\sqrt{50}) = .766 \pm .345 \rightarrow (.421,\ 1.111)$

Natural Gas: $.508 \pm 2.635(.753/\sqrt{50}) = .508 \pm .282 \rightarrow (.226,\ .790)$

Coal: $.438 \pm 2.635(.414/\sqrt{50}) = .438 \pm .155 \rightarrow (.283,\ .593)$

Nuclear: $.161 \pm 2.635(.207/\sqrt{50}) = .161 \pm .076 \rightarrow (.085,\ .237)$

Total: $1.873 \pm 2.635(1.978/\sqrt{50}) = 1.873 \pm .738 \rightarrow (1.135,\ 2.611)$

Petroleum – Natural Gas: $.258 \pm 2.635(.392/\sqrt{50}) = .258 \pm .146 \rightarrow (.112,\ .404)$

(b) Large sample 95% simultaneous T^2 intervals for the indicated means follow. Multiplier is $\sqrt{\chi_4^2(.05)} = \sqrt{9.49} = 3.081$

Petroleum: $.766 \pm 3.081(.925/\sqrt{50}) = .766 \pm .404 \rightarrow (.362,\ 1.170)$

Natural Gas: $.508 \pm 3.081(.753/\sqrt{50}) = .508 \pm .330 \rightarrow (.178,\ .838)$

Coal: $.438 \pm 3.081(.414/\sqrt{50}) = .438 \pm .182 \rightarrow (.256,\ .620)$

Nuclear: $.161 \pm 3.081(.207/\sqrt{50}) = .161 \pm .089 \rightarrow (.072,\ .250)$

Total: $1.873 \pm 3.081(1.978/\sqrt{50}) = 1.873 \pm .863 \rightarrow (1.010,\ 2.736)$

Petroleum – Natural Gas: $.258 \pm 3.081(.392/\sqrt{50}) = .258 \pm .171 \rightarrow (.087,\ .429)$

Since the multiplier, 3.081, for the 95% simultaneous T^2 intervals is larger than the multiplier, 2.635, for the Bonferroni intervals and everything else for a given interval is the same, the T^2 intervals will be wider than the Bonferroni intervals.

5.31 **(a)** The power transformation $\hat{\lambda}_1 = 0$ (i.e. logarithm) makes the duration observations more nearly normal. The power transformation $\hat{\lambda}_2 = -0.5$ (i.e. reciprocal of square root) makes the man/machine time observations more nearly normal. (See Exercise 4.41.) For the transformed observations, say $y_1 = \ln x_1$, $y_2 = 1/\sqrt{x_2}$ where x_1 is duration and x_2 is man/machine time,

$$\bar{\mathbf{y}} = \begin{bmatrix} 2.171 \\ .240 \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} .1513 & -.0058 \\ -.0058 & .0018 \end{bmatrix} \qquad \mathbf{S}^{-1} = \begin{bmatrix} 7.524 & 23.905 \\ 23.905 & 624.527 \end{bmatrix}$$

The eigenvalues for **S** are $\lambda_1 = .15153$, $\lambda_2 = .00160$ with corresponding eigenvectors $\mathbf{e}_1' = [.99925 \quad -.03866]$, $\mathbf{e}_2' = [.03866 \quad .99925]$. Beginning at center $\bar{\mathbf{y}}$, the axes of the 95% confidence ellipsoid are

$$\text{major axis:} \quad \pm\sqrt{\lambda_1}\sqrt{\frac{2(24)}{25(23)}F_{2,23}(.05)}\,\mathbf{e}_1 = \pm.208\mathbf{e}_1$$

$$\text{minor axis:} \quad \pm\sqrt{\lambda_2}\sqrt{\frac{2(24)}{25(23)}F_{2,23}(.05)}\,\mathbf{e}_2 = \pm.021\mathbf{e}_2$$

The ratio of the lengths of the major and minor axes, .416/.042 = 9.9, indicates the confidence ellipse is elongated in the $\mathbf{e}_1$ direction.

(b) $t_{24}(.05/2(2)) = 2.391$, so the 95% confidence intervals for the two component means (of the transformed observations) are:

$$\bar{y}_1 \pm t_{24}(.0125)\sqrt{s_{11}} = 2.171 \pm 2.391\sqrt{.1513} = 2.171 \pm .930 \rightarrow (1.241,\ 3.101)$$

$$\bar{y}_2 \pm t_{24}(.0125)\sqrt{s_{22}} = .240 \pm 2.391\sqrt{.0018} = .240 \pm .101 \rightarrow (.139,\ .341)$$