Example 6.4.5 Let

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \tag{6.74}$$

In the notation of (6.68),

$$a_j = 1, \qquad b_j = 2, \qquad c_j = 1, \qquad \text{all } j$$

Write

$$A = LU = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \alpha_2 & 1 & 0 & 0 & 0 \\ 0 & \alpha_3 & 1 & 0 & 0 \\ 0 & 0 & \alpha_4 & 1 & 0 \\ 0 & 0 & 0 & \alpha_5 & 1 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 & c_1 & 0 & 0 & 0 \\ 0 & \beta_2 & c_2 & 0 & 0 \\ 0 & 0 & \beta_3 & c_3 & 0 \\ 0 & 0 & 0 & \beta_4 & c_4 \\ 0 & 0 & 0 & 0 & \beta_5 \end{bmatrix}$$

From (6.71), $\beta_1 = 2$ and

$$\alpha_j = \frac{1}{\beta_{j-1}}, \qquad \beta_j = 2 - \alpha_j, \qquad j = 2, 3, 4, 5$$

This leads to

$$A = LU = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 1 & 0 & 0 \\ 0 & 0 & \frac{3}{4} & 1 & 0 \\ 0 & 0 & 0 & \frac{4}{5} & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 & 0 \\ 0 & 0 & \frac{4}{3} & 1 & 0 \\ 0 & 0 & 0 & \frac{5}{4} & 1 \\ 0 & 0 & 0 & 0 & \frac{6}{5} \end{bmatrix}$$