Math 34, Second Midterm Review Questions
Fall, 2003

Part I. Material from before the first midterm

Review the material from before the first midterm using the first review sheet and the midterm itself.

Part II. Material since the first midterm

1. State the existence and uniqueness theorem for 2nd order linear ordinary differential equations (Theorem 3.2.1).
2. What is the basic structure of the set of solutions to a 2nd order linear *homogeneous* ordinary differential equation? (Answer: The set of solutions is a two dimensional vector space.)
3. What is meant by *linear independence* of a set $\{f_1, f_2, \ldots, f_s\}$ of functions? (Answer: The set $\{f_1, f_2, \ldots, f_s\}$ of functions is *linearly independent* if the only solution $(c_1, c_2, \ldots, c_s)$ to the equation
$$c_1 f_1 + c_2 f_2 + \cdots + c_s f_s = 0$$
is $c_1 = c_2 = \cdots = c_s = 0$. Equivalently, if
$$c_1 f_1(t) + c_2 f_2(t) + \cdots + c_s f_s(t) = 0$$
for all t, then $c_1 = c_2 = \cdots = c_s = 0$.)
4. What does it mean that a set $\{f_1, f_2, \ldots, f_s\}$ of functions is a *spanning set* for a vector space V of functions? (Answer: The functions $f_1, f_2, \ldots, f_s$ are elements of V and for any element g of V, there exist numbers $c_1, c_2, \ldots, c_s$ such that
$$c_1 f_1 + c_2 f_2 + \cdots + c_s f_s = g.$$
5. What does it mean for a set of functions $\{f_1, f_2, \ldots, f_s\}$ to be a *basis* of a vector space V of functions? (Answer: It means that $\{f_1, f_2, \ldots, f_s\}$ is linearly independent and a spanning set for V. Remark: If V is known to have a certain dimension s then a set of s functions in V is a basis of V $\Leftrightarrow$ the set is linearly independent $\Leftrightarrow$ the set is spanning for V.)
6. What is meant by a *fundamental family* of solutions to a 2nd order linear homogeneous ordinary differential equation? (Answer: A set of (two) solutions to a 2nd order linear homogeneous ordinary differential equation is a fundamental family if it is a *basis* of the vector space of all solutions. Remark: Since the vector space of solutions is known to be two–dimensional, a set of two solutions is a basis $\Leftrightarrow$ it is linearly independent.)

7. What is the Wronskian and what is it good for? (Answer: The Wronskian of a pair of functions f, g is the determinant of the matrix
$$\begin{pmatrix} f(t) & g(t) \\ f'(t) & g'(t) \end{pmatrix},$$
i.e. $W(f,g)(t) = f(t)g'(t) - f'(t)g(t)$. The Wronskian can be used to test for linear independence of the pair of functions: namely if there is at least one t_0 such that $W(f,g)(t_0) \neq 0$ then f, g are linearly independent. Remark: If f, g are both solutions to a certain second order linear homogenous differential equation then either $W(f,g)(t) \neq 0$ for *all* t or $W(f,g)(t) = 0$ for *all* t.)
8. Suppose f, g are two solutions to a second order linear homogenous differential equation. How can one tell if they form a fundamental family of solutions? (Answer: It suffices to check that they are linearly independent. Equivalently, it suffices that $W(f,g)(t) \neq 0$ for some (and hence all) t.)
9. Show that the following pairs of functions are linearly independent:
   (a) $f(t) = t^3 - 5t$, $g(t) = t^3 + 5t$
   (b) $f(t) = e^{2t}$, $g(t) = t^3 e^{2t}$
10. Find the solution to the initial value problem $16y'' - 8y' + 145y = 0$, $y(0) = 1, \quad y'(0) = -1$.
11. Find the general solution to the differential equation $y'' - 2y' - 3y = 3e^{2t}$
12. Find the general solution to the differential equation $y'' - 2y' - 3y = 3te^{3t}$
13. What do you do about complex roots of the characteristic equation for a linear constant coefficient 2nd order ODE?
14. What do you do about repeated roots of the characteristic equation for a linear constant coefficient 2nd order ODE?
15. What sort of non-homogeneous terms (i.e. right hand sides) can you deal with by the method of undetermined coefficients? For each type, what is the form of the particular solution that you try? How does this depend on the nature of the roots of the characteristic equation for the associated homogeneous equation?