Definition 0.1. Let D be any set and $f : D \to \mathbb{R}$ a function. An element $\alpha \in D$ is a (global or absolute) *maximum* for f on D if for all $x \in D$, $f(\alpha) \geq f(x)$. The corresponding value of the function $f(\alpha)$ is called the *maximum value*. *Minimum* is defined similarly.

Definition 0.2. Let $f : D \to \mathbb{R}$ as above. The function f is said to be *bounded above* on D if there exists some number M such that $f(x) \leq M$ for all $x \in D$. The function f is said to be *bounded below* on D if there exists some number m such that $f(x) \geq m$ for all $x \in D$. The function f is said to be *bounded* if it is bounded both above and below.

Now let D be a subset of R (the real line) or $\mathbb{R}^2$ (the plane) or $\mathbb{R}^3$ (3–dimensional space). There is a notion of distance on D. Denote the distance between two point α, β in D by $d(\alpha, \beta)$.

Definition 0.3. A *neighborhood* in D of a point $\alpha \in D$ is a set of the form:

$$\{x \in D : d(\alpha, x) < \delta\},$$

for some $\delta > 0$. In words, the set of all points in D which are closer than distance δ to α.

Definition 0.4. Let D be a subset of $\mathbb{R}$ or $\mathbb{R}^2$ or $\mathbb{R}^3$ and let $f : D \to \mathbb{R}$. A point $\alpha \in D$ is a *local or relative max* for f if there exists some neighborhood N of α such that $f(\alpha) \geq f(x)$ for all $x \in N$.

According to our definitions, every global max is also a local max, but not the other way around. This is not true with the definitions in the text. Ours are the standard and conventional definitions.

Theorem 0.5 (Extreme Value Theorem)**.** *Let $I = [a, b]$ be a closed and finite length interval in $\mathbb{R}$, and let $f : I \to \mathbb{R}$ be a continuous function. Then f is bounded and has a max and min in I.*

Theorem 0.6 (Fermat's theorem)**.** *Let I be an interval of any type and $f : I \to \mathbb{R}$ any function. If γ is an interior point of I (that is, not an endpoint), and γ is a local min of f, and f is differentiable at γ, then $f'(\gamma) = 0$.*

Conseqence: Let $f : [a, b] \to \mathbb{R}$ be a continuous function. By the extreme value theorem, it has a max and min. Consider a max or min γ. Only the following are possible: (1) γ is an endpoint. (2) γ is an interior point and f is not differentiable at γ. (3) γ is an interior point, f is differentiable at γ and $f'(\gamma) = 0$. Thus to find the extreme points (max and min) we only have to consider the endpoints of the interval and points in the interior where either the function is not differentiable or where the derivative is zero.