

Math 16, First Midterm Exam
March 2, 2000

Instructions: This exam has 8 questions and 7 pages. The relative weight of the various questions is indicated. Do all the exercises, writing your answers in this exam booklet. Show your work. Your work will be judged for correctness, completeness, clarity and orderliness. Put your final answer to each question in a box so that it can be located easily.

1. (20 points) A bacterial population grows according to the law:

$$P(t) = P(0)(1.3)^t,$$

where t is measured in hours.

- (a) At what time will the population be three times the initial value $P(0)$?
- (b) At what time will the population be six times the initial value $P(0)$?

2. (20 points) A drug is given in daily doses of M mg. In any one day period, 50% of the amount of drug present in the body is eliminated. Find a formula for the amount of drug present after the n th dose (in terms of M), and find the appropriate value of M if a steady state (limiting) amount of 400 mg is desired.

3. (15 points) Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = x^2 + \frac{3}{7}x + 16.$$

- (a) Find the range Y of the function.
- (b) Find a suitable domain D such that the restriction of f to D is one-to-one, with range R . (That is, $f : D \rightarrow Y$ is one-to-one and onto).

4. (10 points) Find the inverse of the function $f(x) = \frac{x}{1+x}$.

5. (20 points) Show by induction that for all natural numbers n ,
- $$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

6. (10 points) Write the negation of the (true) sentence: *This test is really easy, and this is the best mathematics course I've ever taken.*

7. (10 points) Let $f(x) = \frac{x}{1+x}$ and $g(x) = x^2 + 2$. Find $f \circ g$ and $g \circ f$.
You need not do any simplification.

8. (15 points) Assume that the temperature in Iowa City during a certain period in January follows a sinusoidal daily oscillation, with average value 25 degrees F., amplitude 10 degrees F., period 24 hours (of course), and the daily maximum temperature occurring at 1 pm. Find a formula for the temperature as a function of time t , where t is measured in hours, and $t = 0$ represents midnight of the first day of the period during which the description is valid. Draw a graph of the temperature function.