Mathematics 121 Review I
March, 2006

In this review, references to the text are to the *latest posted versions.* This does not mean that you have to print out the latest versions.

Definitions and theorem statements you should know:

1. Vector space, linear independence, linear combination, spanning, span, basis, subspace, quotient space, quotient homomorphism, finite dimensional, dimension, dual vector space, dual basis, matrix of a linear transformation with respect to a basis, similarity of matrices and linear transformations.
2. Module, linear independence, generating set, basis, free module, torsion, torsion module, torsion–free module, submodule, quotient module, quotient homomorphism.
3. Multilinear map, symmetric, skew–symmetric, and alternating multilinear maps, determinant.
4. Existence and uniqueness of invariant factor decomposition for finitely generated module over a PID.
5. Existence and uniqueness of elementary divisor decomposition for finitely generated module over a PID.

Theorems you should be able to prove:

1. Any of the homomorphism theorems for vector spaces or modules. (It's ok to quote the corresponding theorem for groups and show relevant maps are linear, resp. R–linear.
2. Any vector space has a basis. If V has a finite spanning set, then it has a finite basis.
3. Any two bases of a finite dimensional vector space have the same cardinality.
4. The universal property of a basis. (Prop. 3.3.32 in latest posted version.)
5. Any subspace of a vector space has a complement. (It's ok if you just know this for finite dimensional spaces. Use an argument with bases, or use a right inverse for the quotient map, $V \to V/W$, as in Prop. 3.3.33–34.)
6. A finite dimensional vector space and its dual space have the same dimension. A finite dimensional vector space is naturally isomorphic to its second dual space.
7. Characterization of determinant (Cor. M.3.8).
8. Multiplicativity of determinant.
9. A linear transformation on a finite dimensional vector space V over K determines a finitely generated torsion $K[x]$–module structure on V.
10. Two linear transformations are similar if, and only if, they determine isomorphic $K[x]$– modules.
11. Existence and uniqueness of RCF (based on the invariant factor decomposition).
12. Existence and uniqueness of JCF of a matrix $A \in \mathrm{Mat}_n(K)$ (based on the elementary divisor decomposition, assuming the characteristic polynomial splits in $K[x]$.)

13. Relation between characteristic polynomial and invariant factors of a matrix. The Cayley-Hamilton Theorem.

Be able to compute:

1. Change of basis for a linear transformation.
2. Jordan canonical form of a matrix.
3. Rational canonical form of a matrix.
4. Passage from RCF to JCF or vice versa.

Go through Oscar's handouts and review on RCF and JCF.