YOUR NAME:

YOUR TA'S NAME:

Math 32, FIRST MIDTERM EXAM
March 26, 2003

Scores

1	2	3	4	5	total

Instructions:

(1) **Please write your name and the name of your TA prominently at the top of this front page.!**

(2) This exam has 5 questions and 6 pages. The relative weight of the various questions is indicated. Do all the exercises, writing your answers in this exam booklet. Show your work. Your work will be judged for correctness, completeness, clarity and orderliness. Put your final answer to each question in a box so that it can be located easily.

(3) When you are finished, please turn in your exam paper to your own TA.

(1) (20 points)

(a) Find an equation of the line (in x–y coordinates) passing through the point $(-1, 3)$ and *perpendicular* to the vector

$$\mathbf{A} = \begin{bmatrix} -4 \\ 5 \end{bmatrix}.$$

If you would like, find the equation first in local (dx–dy coordinates), but then translate into x–y coordinates.

(b) Find parametric equations for the line in the x–y plane containing the point $(-1, 3)$ and *parallel* to the vector

$$\mathbf{A} = \begin{bmatrix} -4 \\ 5 \end{bmatrix}.$$

(2) (20 points)

(a) Find the gradient $\mathbf{G} = \nabla f(1, 4)$ of the function

$$f(x, y) = x^2 - 3xy$$

at the point $(1, 4)$.

(b) Find an equation in x–y–z coordinates for the plane tangent to the surface

$$z = x^2 - 3xy$$

at the point $(1, 4, -11)$.

If you want, find the equation first in local (dx–dy–dz coordinates), but then translate into x–y–z coordinates.

(3) (30 points)

(a) Find an equation for the tangent line to the level curve

$$x^2 - 3xy = -11$$

at the point $(1, 4)$.

(b) Find the instantaneous rate of change of the function

$$f(x, y) = x^2 - 3xy$$

as one moves from the point $(1, 4)$ in the direction parallel to the vector $\mathbf{X} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

(c) In what direction $\mathbf{u}$ is the directional derivative $D_{\mathbf{u}}f(1, 4)$ *minimum*? Find the value of the directional derivative in that direction.

(4) (20 points)

(a) Find a two dimensional vector which is perpendicular to the line

$$2x + 4y = 7$$

.

(b) Find a three dimensional vector which is perpendicular to the plane

$$3x + 4y - 5z = 8.$$

(c) Consider the tangent plane to the surface

$$z = x^2 - 3xy$$

at the point $(1, 4, -11)$. Find a three dimensional vector which is perpendicular to this tangent plane. (Note: you already computed the tangent plane in problem 2b — no need to do it again!)

(5) (20 points) Consider the function of 3 variables

$$F(x, y, z) = z^2 - x^2 + 3xy,$$

and its level surface

$$z^2 - x^2 + 3xy = 20.$$

The point $A = (1, 4, 3)$ is on this surface.

(a) Compute the gradient of F at $A = (1, 4, 3)$,

$$\nabla F(1, 4, 3).$$

(b) Use the answer to part (a) to write down an equation for the tangent plane to the surface at $A = (1, 4, 3)$.