REVIEW QUESTIONS FOR FIRST MIDTERM

(1) Let $\mathbf{a} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$.

  (a) Compute $\mathbf{a} \times \mathbf{b}$.

  (b) One can write $\mathbf{a} = \mathbf{a_1} + \mathbf{a_2}$, where $\mathbf{a_1}$ is parallel to $\mathbf{b}$ and $\mathbf{a_2}$ is perpendicular to $\mathbf{b}$. Compute $\mathbf{a_1}$ and $\mathbf{a_2}$.

(2) How can you compute the volume of the parallelopiped determined by three vectors? How can you determine the area of the parallelogram determined by two vectors (in three dimensional space)?

(3) What are the properties of the cross product?

(4) What is an equation for the plane containing the point $(1, 2, 4)$ and perpendicular to the vector $\begin{bmatrix} 2 \\ 5 \\ -2 \end{bmatrix}$?

(5) What is an equation for the tangent plane to the function $1 - x^3y^4$ at $(1/2, 2)$? What is a vector perpendicular to the tangent plane?

(6) What is the directional derivative of $f(x, y) = 1 - x^3y^4$ in the direction of the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ at the point $(1/2, 2)$? In what direction $\mathbf{u}$ is the directional derivative $D_{\mathbf{u}}f(1/2, 2)$ maximum, and what is the value of the maximum directional derivative?

(7) Suppose you can measure the radius and height of a cylinder with $\pm 1\%$ accuracy. Your measurements are $r = 4$ and $h = 5$ in appropriate units. What is the maximum expected error in the computed value of volume, $V = \pi r^2 h$, and what is the maximum expected relative error (ratio of the error to the computed value)?

(8) What does it mean for a function $f(x, y)$ to be differentiable (or to admit a tangent plane approximation) at a point (x_0, y_0). Answer: Let

$$\lambda(x, y) = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0),$$

and

$$\varepsilon(x, y) = f(x, y) - \lambda(x, y).$$

Then $f(x, y)$ is differentiable at (x_0, y_0) if

$$\lim_{(x,y)\to(x_0,y_0)} \frac{\varepsilon(x, y)}{\sqrt{(x - x_0)^2 + (y - y_0)^2}} = 0.$$

(9) Give an example of a function of two variables and a point where the function is not differentiable.

(10) What is smoothness and what is it good for? Answer: A function of two variables is said to be smooth in a region in the plane if the function and its two partial derivatives are continuous in the region. Smoothness is a good condition because it is relatively easy to check and it implies differentiability of the function at all points in the region where the function is smooth. (We will soon see other implications of smoothness.)

(11) Study Professor Stroyan's collection of review sheets. I am going to look at them for ideas when I prepare the exam.