YOUR NAME:

YOUR TA'S NAME:

Math 32, FINAL EXAM
MAY 10, 2004

Scores

1	2	3	4	5	6	total

Instructions:

(1) **Please write your name and the name of your TA prominently at the top of this front page.!**

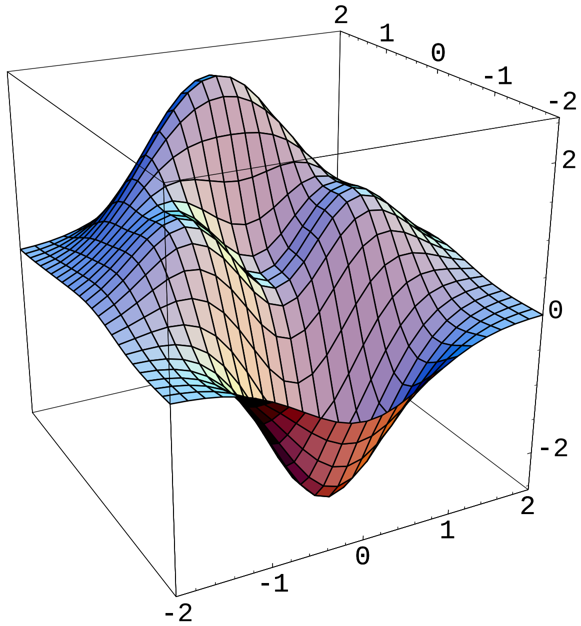
(2) This exam has 6 questions and 8 pages. The relative weight of the various questions is indicated. Do all the exercises, writing your answers in this exam booklet. Show your work. Your work will be judged for correctness, completeness, clarity and orderliness. Put your final answer to each question in a box so that it can be located easily.

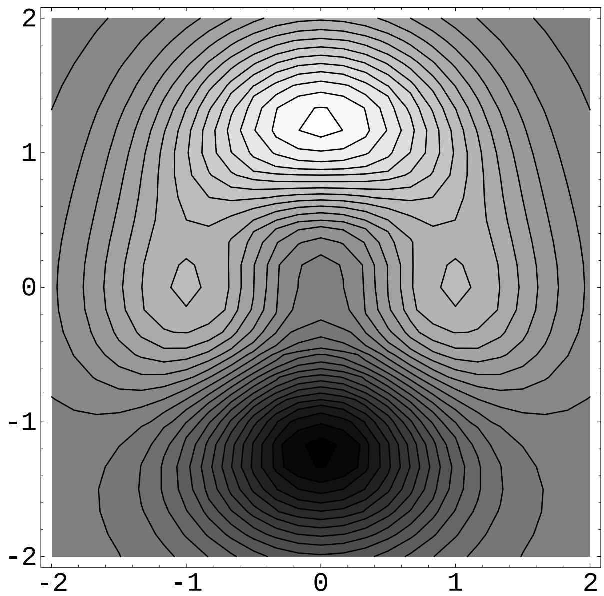
(3) When you are finished, please turn in your exam paper to your own TA.

(1) (10 points) Consider the function of two variables

$$f(x,y) = 3\,e^{-x^2-y^2}\,\left(x^2+2\,y^3\right).$$

Here are 3D and contour graphs of this function.





The partial derivatives of f are

$$\frac{\partial f}{\partial x}(x,y) = -6\,e^{-x^2-y^2}\,x\,\left(-1+x^2+2\,y^3\right)$$

and

$$\frac{\partial f}{\partial y}(x,y) = -6\,e^{-x^2-y^2}\,y\,\left(x^2-3\,y+2\,y^3\right).$$

Mathematica gives the solutions to the equations

$$\frac{\partial f}{\partial x}(x,y) = 0, \quad \frac{\partial f}{\partial y}(x,y) = 0,$$

as

$$\{x \to -1, y \to 0\},$$
$$\{x \to 0, y \to 0\},$$
$$\{x \to 0, y \to -\sqrt{\frac{3}{2}}\},$$
$$\{x \to 0, y \to \sqrt{\frac{3}{2}}\},$$
$$\{x \to 1, y \to 0\},$$
$$\{x \to \frac{-5}{3\sqrt{3}}, y \to \frac{1}{3}\},$$
$$\{x \to \frac{5}{3\sqrt{3}}, y \to \frac{1}{3}\}$$

Find the max and min of the function.

(2) (20 points)

(a) Find an equation for the plane P which is perpendicular to the vector $\mathbf{N} = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}$ and which passes through the point $A = (1 - 1, 2)$.

(b) Find the distance from the origin $O = (0, 0, 0)$ to P. (*Hint*: The distance is the length of the projection of $\vec{AO}$ onto the vector $\mathbf{N}$.)

(3) (20 points)

(a) Find the gradient $\mathbf{G} = \nabla f(1,4)$ of the function

$$f(x,y) = x^2 - 3xy$$

at the point $(1,4)$.

(b) Find an equation in x–y–z coordinates for the plane tangent to the surface

$$z = x^2 - 3xy$$

at the point $(1,4,-11)$.

(c) Find the directional derivative of f at the point $(1,4)$ in the direction of the vector $\begin{bmatrix}1\\2\end{bmatrix}$.

(4) (20 points) Consider the vector field $\mathbf{F}(x,y) = \begin{bmatrix} x^2 - y \\ x + y^2 \end{bmatrix}$. Let C be the straight line path from from $(-2,1)$ to $(1,2)$.

(a) Find a parametrization $X(t)$ of C.

(b) Compute the flow $\int_C \mathbf{F}(\mathbf{x}) \cdot d\mathbf{x}$ along the curve C. (*Hint*: First write down the "abstract" formula for the integral, valid for any vector field, any curve, and any parametrization. Then put in the particular data for this exercise. Finally, compute.)

(c) Compute the flow $\int_C \mathbf{F}(\mathbf{x}) \cdot \mathbf{n} ds$ across the curve C. (*Hint*: Same hint as in part (b).)

(5) (20 points) Consider the vector field $\mathbf{F}(x,y) = \begin{bmatrix} x^2 - y \\ x + y^2 \end{bmatrix}$. Let C be the unit circle traversed in the counterclockwise direction.

(a) Use Green's theorem to calculate the flow $\int_C \mathbf{F}(\mathbf{x}) \cdot d\mathbf{x}$ around the curve C. (*Hint*: First write down the "abstract" version of Green's theorem, valid for any vector field, and any simple closed curve. Then put in the particular data for this exercise. Finally, compute.)

(b) Use Green's theorem to calculate the flow $\int_C \mathbf{F}(\mathbf{x}) \cdot \mathbf{n} ds$ out of the curve C. (*Hint*: Same hint as in part (a).)

(6) (20 points) Consider the series

$$f(x) = \sum_{k=0}^{\infty} \frac{\sin(7kx)}{3^k}$$

(a) Estimate the error if the series is truncated after 100 terms. That is, estimate

$$\left| f(x) - \sum_{k=0}^{100} \frac{\sin(7kx)}{3^k} \right| = \left| \sum_{k=101}^{\infty} \frac{\sin(7kx)}{3^k} \right|.$$

(b) Find n so that the error

$$\left| f(x) - \sum_{k=0}^{n} \frac{\sin(7kx)}{3^k} \right|$$

is less than $\frac{1}{1000}$, independent of x.