CORRECTIONS TO THE TALK"REPRESENTATIONS OF THE AUTOMORPHISM GROUP OF A PRO-FREE GROUP" BY TED CHINBURG

After the talk, I realized that I'd misquoted some of the main results of Grunewald and Lubotzky. In the notation of the talk, they first define the linear algebraic group G over $\mathbb{Q}$ to be the $\mathbb{Q}[H]$-linear automorphisms of the module $\mathbb{Q} \otimes_{\mathbb{Z}} \overline{R}$, where $\overline{R}$ is the maximal abelian quotient of the relation group R defined by the exact sequence

$$1 \longrightarrow R \longrightarrow F_n \longrightarrow H \longrightarrow 1.$$

Define the algebraic group $G_{\mathbb{Z}}$ over $\mathbb{Z}$ to be $\mathrm{Aut}_{\mathbb{Z}[H]}(\overline{R})$. Then $G_{\mathbb{Z}}$ has base change G to $\mathbb{Q}$. Grunewald and Lubotzky define $G^1_{\mathbb{Z}}$ to be the subgroup of $G_{\mathbb{Z}}$ which is the kernel of all homomorphisms from $G_{\mathbb{Z}}(\mathbb{C})$ to $\mathbb{C}^*$ which are defined over $\mathbb{Q}$. Such homomorphisms come about from writing G as a finite product $\prod_i \mathrm{GL}_{n_i}(D_i)$ in which each D_i is a division algebra with center a number field K_i. To define $G^1_{\mathbb{Z}}$ inside $G_{\mathbb{Z}}$, it is then sufficient to take the intersection of $G_{\mathbb{Z}}$ with the kernels of the homomorphisms $G \longrightarrow K_i^*$ resulting from the i^{th} projection $G \longrightarrow \mathrm{GL}_{n_i}(D_i)$ followed by the reduced norm $\mathrm{GL}_{n_i}(D_i) \longrightarrow K_i^*$. One needs to be careful about how Grunewald and Lubotzky define this reduced norm; see [1, §2.3]. The group $G^1_{\mathbb{Z}}$ is in general smaller than the group I mentioned in my talk consisting of the subgroup of $G_{\mathbb{Z}}$ which induces the trivial automorphism of the top exterior power of $\overline{R}$ over $\mathbb{Z}$. The main result of Grunewald and Lubotzky which I quoted, Theorem 1.4 of [1], has the hypothesis that $n \geq 4$ and that the homomorphism $F_n \longrightarrow H$ sends some generator of F_n to the identity element of H.

In the talk, I mentioned the example in which H is a cyclic group of prime order p. When $n = 2$, the group G will be isomorphic to $\mathrm{GL}_2(\mathbb{Q}) \times \mathbb{Q}(\zeta_p)^*$, as stated in the talk. The group $G^1_{\mathbb{Z}}(\mathbb{Z})$ will be commensurable with $\mathrm{SL}_2(\mathbb{Z}) \times \{1\}$ since the reduced norm on the factor $\mathbb{Q}(\zeta_p)^*$ of G is the identity map $\mathbb{Q}(\zeta_p)^* \longrightarrow \mathbb{Q}(\zeta_p)^*$. Thus the group $G^1_{\mathbb{Z}}(\mathbb{Z})$ is considerably smaller than the group I described in my talk pertaining to the Grunewald-Lubotzky results. Note that this group has to do with automorphisms of the discrete free group F_n, and [1, Theorem 1.4] does not apply because $n = 2$. For more discussion of the $n = 2$ case when H is cyclic, see [1, §6].

The new results in my talk had to do with the automorphisms of the profinite completion $\hat{F}_n$ of F_n. In the profinite case, the representations of a subgroup of finite index in $\mathrm{Aut}(\hat{F}_n)$ will have much larger image since there are no reduced norm one restrictions on elements of the image. In particular, the results in the profinite case when $n = 2$ and H is cyclic of order p show that the image of the resulting representation is commensurable to

$$\mathrm{GL}_2(\hat{\mathbb{Z}}) \times \hat{\mathbb{Z}}[\zeta_p]^*.$$

References

[1] Grunewald, F and Lubotzky, A.: "Linear representations of the automorphism group of a free group," GAFA, Geom. funct. anal. Vol. 18 (2009), p. 1564 - 1608.