CONFERENCE ON GEOMETRIC METHODS IN REPRESENTATION THEORY

UNIVERSITY OF MISSOURI – COLUMBIA

NOVEMBER 23-25, 2019

Conference Schedule

All talks are in STRICKLAND HALL, rooms Strickland 114 and Strickland 109.
All non-parallel talks are in 114 STRICK.

	Saturday, November 23
8:45-9:00	WELCOME (114 STRICK)
9:00-10:00	**Angeleri Hügel** (keynote) (114 STRICK) *The lattice of ring epimorphisms of a finite dimensional algebra.*
10:00-10:15	Questions
10:15-10:45	**Todorov** (114 STRICK) *Continuous quivers of type A.*
10:45-11:25	Questions COFFEE
11:25-12:15	**Yakimov** (expository) (114 STRICK) *Noncommutative tensor triangular geometry.*
12:15-2:30	Questions LUNCH BREAK
2:30-3:00	**Derksen** (114 STRICK) *Invariant theory and wheeled PROPs.*
3:00-3:10	Questions
3:10-3:40	**Igusa** (114 STRICK) *Amalgamation, unamalgamation and the ϕ-dimension conjecture.*
3:40-4:10	Questions COFFEE
4:10-4:40	**Paquette** (114 STRICK) *Finitistic dimension: a reduction technique.*
4:40-4:50	Questions

	Sunday, November 24
9:00-10:00	**Angeleri Hügel** (keynote) (114 STRICK) *The lattice of ring epimorphisms of a finite dimensional algebra.*
10:00-10:40	Questions COFFEE
10:40-11:30	**Schiffler** (expository) (114 STRICK) *Frieze varieties: a characterization of the finite-tame-wild trichotomy.*
11:30-11:45	Questions
11:45-12:15	**Chinburg** (114 STRICK) *Representations of the automorphism group of a pro-free group.*
12:15-2:30	Questions LUNCH BREAK
2:30-3:00	**Lin** (114 STRICK) *Free tree algebras arising from Rota-Baxter extensions.*
3:00-3:10	Questions
3:10-3:30	**Serhiyenko** (114) *Mutation of type D friezes.*
3:30-4:00	Questions COFFEE and move to parallel
4:00-4:20	**Wood** (109) *Module structure of the space of holomorphic polydifferentials.* **Cheung** (114) *Compactification for cluster varieties without frozen variables of finite type.*
4:20-4:30	Questions
4:30-4:50	**Meyer** (109) *Tracking the variety of interleavings.* **Garver** (114) *Reverse plane partitions via quiver representations.*
4:50-5:00	Questions

	Monday, November 25
9:00-9:30	**Thomas** (114 STRICK) *The fundamental theorem of finite semidistributive lattices.*
9:30-10:10	Questions COFFEE and move to parallel
10:10-10:30	**Zhu** (109) *Hopf algebras of discrete representation type.* **Yildirim** (114) *A completion of discrete cluster categories of type A.*
10:30:10:40	Questions
10:40-11:00	**Sen** (109) *Exceptional sequences and trees.* **Nguyen** (114) *Actions of Hopf algebra and stratification of noncommutative prime spectra.*
11:00-11:20	Questions and regroup
11:20-11:50	**Beil** (114 STRICK) *An application of nonnoetherian algebraic geometry to physics.*
11:50-12:00	Questions

Abstracts

Keynote Lectures

Lidia Angeleri Hügel (Università degli Studi di Verona, Italy).

The lattice of ring epimorphisms of a finite dimensional algebra.

The ring epimorphisms starting in a given ring A form a complete lattice which encodes valuable information on A and its representation theory. Aim of the talks is to review the main properties of this lattice and its role in a number of classification results. Along the way, we will discuss the notion of a silting complex and the dual notion of a cosilting complex. We will explain how these concepts are related with ring epimorphisms and torsion pairs. Then we will focus on two main cases: τ-tilting finite algebras and finite dimensional hereditary algebras. The talks will be based on joint work with Frederik Marks, Jorge Vitória, and Michal Hrbek.

Expository Lectures

Milen Yakimov (Louisiana State University).

Noncommutative tensor triangular geometry.

We will describe a general noncommutative version of Balmer's tensor triangular geometry that is applicable to all monoidal triangulated categories. Using insight from noncommutative ring theory, to such a category K, we will associate a topological space, the Balmer spectrum $\mathrm{Spc}(K)$, and prove that it is a universal terminal object for support data for K. In the opposite direction, support data will be used in theorems describing $\mathrm{Spc}(K)$ as a topological space. Throughout the talk we will run a comparison between the commutative and noncommutative sides of the constructions. Examples will be given for the stable module categories of noncocommutative Hopf algebras (such as the quantum Borel algebras and the Benson-Witherspoon Hopf algebras). The talk is based on the original papers of Balmer and a joint work of a Daniel Nakano, Kent Vashaw and the speaker.

Ralf Schiffler (University of Connecticut).

Frieze varieties: a characterization of the finite-tame-wild trichotomy.

This is a joint work with Kyungyong Lee, Li Li, Matt Mills and Alexandra Seceleanu. We introduce a new class of algebraic varieties which we call frieze varieties. Each frieze variety is determined by an acyclic quiver. The frieze variety is defined in an elementary recursive way by constructing a set of points in affine space. From a more conceptual viewpoint, the coordinates of these points are specializations of cluster variables in the cluster algebra associated to the quiver. We give the following new characterization of the finite–tame–wild trichotomy for acyclic quivers in terms of their frieze varieties. An acyclic quiver is representation finite, tame, or wild, respectively, if and only if the dimension of its frieze variety is 0, 1, or > 1, respectively.

Conference Talks

Charlie Beil (University of Graz).
An application of nonnoetherian algebraic geometry to physics.

The framework of nonnoetherian algebraic geometry, as the geometry of algebraic varieties with positive dimensional 'smeared-out' points, arose as a tool to understand the representation theory of a class of quiver algebras called dimer algebras. In this talk I will describe an application of this strange geometry to physics: if the trajectory of an elementary particle is a one-dimensional point of spacetime, rather than a continuum of distinct points, then the particle inherits new properties that may be identified with electric charge and quark color charge. I will also illustrate how the resulting 'nonnoetherian spacetime' serves as a framework to describe quantum nonlocality.

Mandy Cheung (Harvard University).
Compactification for cluster varieties without frozen variables of finite type.

Cluster varieties are blow up of toric varieties. They come in pairs (A, X), with A and X built from dual tori. Compactifications of A, studied by Gross, Hacking, Keel, and Kontsevich, generalize the polytope construction of toric varieties while the compactifications of X, studied by Fock and Goncharov, generalize the fan construction. The conjecture is that the A and the X cluster varieties are mirrors to each other. Together with Tim Magee, we have shown that there exists a positive polytope for the type A cluster varieties which give us a hint to the Batyrev-Borisov construction.

Ted Chinburg (University of Pennsylvania).
Representations of the automorphism group of a pro-free group.

In this talk I will describe work with F. Bleher and A. Lubotzky on representations of the automorphism group $\mathrm{Aut}(\hat{F}_r)$ of a pro-free group $\hat{F}_r$ on some number r of generators. We use a method applied by Grunewald and Lubotzky to the analogous problem when $\hat{F}_r$ is replaced by a discrete free group F_r on r generators. In the pro-finite case, one can show a sharper result about the images of the representations. I will then show how this can be applied to Galois theory. This uses the Belyi embedding of the absolute Galois group $G_{\mathbb{Q}}$ of the rationals into $\mathrm{Aut}(\hat{F}_2)$. We show in particular that the natural action of certain subgroups of $G_{\mathbb{Q}}$ on the Tate modules of Jacobians of covers of $\mathbb{P}^1$ branched at 3 points can be lifted to an action of $\mathrm{Aut}(\hat{F}_2)$.

Harm Derksen (University of Michigan).
Invariant theory and wheeled PROPs.

Wheeled PROPs were introduced by Markl, Merkulov and Shadrin. This framework seems perfect for studying problems related to tensor invariants. As in the category of commutative rings with 1, the initial object and its ideals play a crucial role. I will discuss classify the ideals and prime ideals in the initial object, and discuss applications of wheeled PROP to invariant theory, such as generalizations of a theorem of Procesi on embeddings of trace rings into matrix algebras and a Galois type correspondence of Schrijver for tensor invariants. This is joint work with Visu Makam.

Alexander Garver (University of Michigan).
Reverse plane partitions via quiver representations.

A reverse plane partition is an order reversing map from a poset to the nonnegative integers. We study reverse plane partitions on the so-called minuscule posets. A minuscule poset is defined by choosing a Dynkin diagram and a minuscule vertex of the Dynkin diagram. We show that there is a bijection between reverse plane partitions on a minuscule poset and representations of a Dynkin quiver of the corresponding type all of whose indecomposable summands are supported on the minuscule vertex. This is joint work with Rebecca Patrias and Hugh Thomas.

Kiyoshi Igusa (Brandeis University).
Amalgamation, unamalgamation and the ϕ-dimension conjecture.

The ϕ-dimension conjecture states that, for any artin algebra Λ, there is a uniform bound on the ϕ-dimension of the finitely generated Λ-modules. For modules of finite projective dimension, the projective dimension is equal to the ϕ-dimension. Therefore, ϕ dim-$\Lambda \leq$ findim-Λ. Amalgamation is the process by which triangular quivers (with no relations) are assembled to give the Jacobian algebra of a plabic diagram (planar bicolored graph). Eric Hanson, Gordana Todorov and I are working on the details of how amalgamation and unamalgamation work for quivers with relations. In this talk I will explain how the investigation of amalgamation leads to a counterexample to the ϕ-dimension conjecture.

Zongzhu Lin (Kansas State University).
Free tree algebras arising from Rota-Baxter extensions.

V. Jones used subfactors to construct towers of algebras, which turned out to be closely related to Hecke algebras and led to the discovery of Jones polynomials. We mimic the construction for Rota-Baxter extensions to construct a tower of finite dimensional algebras. This turns out to be a deformation of free-tree monoid algebra considered by A. Ayyer, A. Schilling, B. Steinberg, N. Thiéry in their study of Morkov chains of R-trivial monoids. The additional parameters help to describe certain combinatorial properties of basis in terms of plane diagrams in the analogous fock space, in the same manner as partitions (young diagrams) are used in computations of basis of fock spaces.

David Meyer (Smith College).
Tracking the variety of interleavings.

In topological data analysis, representations of incidence algebras (persistence modules) are used to distinguish legitimate topological properties of a data set from noise. Interleavings between these representations feature prominently in this analysis since they are used to define a metric structure on the collection of persistence modules. One can show that for any $\epsilon > 0$, the collection of ϵ-interleavings between two fixed persistence modules admits the structure of an affine variety. In this talk we investigate how this variety changes with the value of the parameter ϵ.

Bach Nguyen (Temple University).
Actions of Hopf algebra and stratification of noncommutative prime spectra.

Let H be a Hopf algebra, and A be any associative unital algebra. We will discuss how the actions of H on A can be used to study prime ideals in A and ultimately provide effective machinery to solve many difficult ring theoretic problems. In particular, when H is cocommutative, we give an explicit description of the "H-strata" in prime spectrum of A and generalize previous results of Goodearl–Letzter and Lorenz. If time permits, we will also discuss our recent results on the locally finiteness of adjoint representation of H. This is a joint work with M. Lorenz and R. Yammine.

Charles Paquette (Royal Military College of Canada).
Finitistic dimension: a reduction technique.

I will discuss recent results obtained with Diego Bravo. We start with an Artin algebra A and we consider the possibility of embedding A into a larger algebra B (with one more rank) such that under a local condition for B, if the finitistic dimension of B is finite, then that of A is. We will explain how this could provide an interesting tool for studying the famous finitistic dimension conjecture, and we will give applications of our result.

Emre Sen (University of Iowa).
Exceptional sequences and trees.

Exceptional sequences of quiver representations have a rich combinatorics. For type A, the number of exceptional sequences is equal to the number of all rooted labeled forests. Hence we expect to give combinatorial models to exceptional sequences of uniserial modules by using trees. We verify this for linear radical square zero Nakayama algebras: the number of complete exceptional sequences is equal to the number of rooted labeled forests of height at most one.

Khrystyna Serhiyenko (University of Kentucky).
Mutation of type D friezes.

Frieze is an array of positive integers satisfying certain rules. Friezes of type A were first studied by Conway and Coxeter in 1970's, but they gained fresh interest in the last decade in relation to cluster algebras and their categorification in terms of representation theory of cluster-tilted algebras. Thus, a frieze is an arrangement of positive integers on the Auslander-Reiten quiver of a cluster-tilted algebra such that entries coming from a mesh satisfy the so-called diamond rule. In this talk, we will discuss friezes of type D and their mutations. This is joint work with A. Garcia Elsener.

Hugh Thomas (Université du Québec à Montréal).
The fundamental theorem of finite semidistributive lattices.

The goal of this project is to give a combinatorial model for all finite semidistributive lattices. The poset of torsion classes of an algebra, ordered by inclusion, forms a lattice, which is necessarily semidistributive. However, it is easy to write down examples of semidistributive lattices that are not the lattice of torsion classes of any algebra, which seems like an obstacle to using torsion classes to understand semidistributivity. It turns out, though, that any finite semidistributive lattice uniquely defines a kind of combinatorial shadow of a category for which it wants to be the lattice of torsion classes, and, as I will explain, this provides the kind of combinatorial model we were seeking. I will introduce and try to motivate all the necessary concepts from lattice theory. This talk is based on joint work with Nathan Reading and David Speyer, available at arXiv:1907.08050.

Gordana Todorov (Northeastern University).
Continuous quivers of type A.

This is joint work with Kiyoshi Igusa and Job Rock. We consider quivers of continuous type A and prove basic results about pointwise finite dimensional representations, e.g. we prove analogue of the barcode theorem for the continuous quivers of type A with alternating orientations. Starting with this, we define a generalization of the continuous cluster categories. These categories have several new features: continuous clusters and continuous mutations, unlike the original continuous cluster categories.

Adam Wood (University of Iowa).
Module structure of the space of holomorphic polydifferentials.

Let X be a smooth projective curve over a perfect field k and let G be a finite group acting on X. The space of holomorphic m-polydifferentials is defined to be the space of global sections of the m-fold tensor product of the sheaf of relative differentials with itself. This space provides a representation of G and a classical problem is to determine the decomposition of the space of holomorphic polydifferentials as a direct sum of indecomposable representations. We discuss work on this problem in the case when k has prime characteristic p and G has cyclic Sylow p-subgroups. The results have implications for cusp forms on modular curves as well as for the dimension of the tangent space of the deformation functor of curves.

Emine Yildirim (Queen's University).
A completion of discrete cluster categories of type A.

In this talk, I will talk about a joint work with Ba Nguyen and Charles Paquette. We consider a disk with infinitely many marked points and finitely many accumulation points on the boundary. The associated cluster algebras to a similar but more general setting known as infinite rank surface cluster algebras and their combinatorics are studied by A. Felikson and I. Canakci (2017). Our goal is to give a categorification of an inifinite rank surface cluster algebra. I will first talk about the Igusa-Todorov discrete cluster category of type A, and then construct a categorical completion of this category with a geometric approach. This will give us the desired cluster category which is a Hom-finite triangulated category. If time permits, I will also talk about the cluster-tilting subcategories in this setting.

Shijie Zhu (University of Iowa).

Hopf algebras of discrete representation type.

Hopf algebra is an important topic in geometric representation theory. A basic algebra is of finite representation type if there are only finitely many non-isomorphic indecomposable representations. Basic Hopf algebras of finite representation type have been classified by Liu and Li in 2004. As algebras, they are just copies of Nakayama algebras. A pointed coalgebra is of discrete representation type, if there are only finitely many non-isomorphic indecomposable representations for each dimension vector. We are trying to classify pointed Hopf algebras of discrete representation type. We first classify their Ext-quivers as coalgebras, which fall into four types. Then we compute their algebra structures for each case.