Among all finite index subgroups of the modular group, majority of them are noncongruence, that is their group members cannot be described in terms of congruences. It is a commonly believed that the coefficients of genuine holomorphic noncongruence modular forms having algebraic coefficients have unbounded denominators. This conjecture has far-reaching impacts on arithmetic and beyond. In this talk, we will give partial supportive results to this conjecture.

Given a finite index subgroup of the modular group whose modular curve is defined over the field of rationals, under the assumption that the space of (weight at least 2) cusp forms is 1-dimensional, we show that a form in this space with rational Fourier coefficients has unbounded denominators if and only if it is a noncongruence modular form. (Received January 24, 2011)