Suppose $G$ is a locally euclidean group and $M$ is a locally euclidean space, and let

$$\Phi : G \times M \rightarrow M$$

be a continuous action of $G$ on $M$. In his fifth problem Hilbert asks if one then can choose the local coordinates in $G$ and $M$ so that $\Phi$ is real analytic.

When $G = M$ and

$$\Phi : G \times G \rightarrow G$$

is the multiplication in the group $G$ the answer to Hilbert’s question is affirmative, as was proved by Gleason, Montgomery and Zippin.

For the question (1) we prove.

**Theorem.** Let $G$ be a Lie group which acts on a $C^1$ smooth manifold $M$ by a $C^1$ smooth proper action. Then there exists a real analytic structure $\beta$ on $M$, compatible with the given smooth structure on $M$, such that the action of $G$ on $M_\beta$ is real analytic.

Concerning the uniqueness of $\beta$ in Theorem 1 we have (from a paper by the author and Marja Kankaanrinta).

**Theorem.** Let $M$ and $N$ be real analytic proper $G$-manifolds, where $G$ is a linear Lie group. Suppose that $M$ and $N$ are $G$-equivariantly $C^1$ diffeomorphic. Then $M$ and $N$ are $G$-equivariantly real analytically isomorphic.