An Interactive SMT Tactic in Coq using Abductive Reasoning

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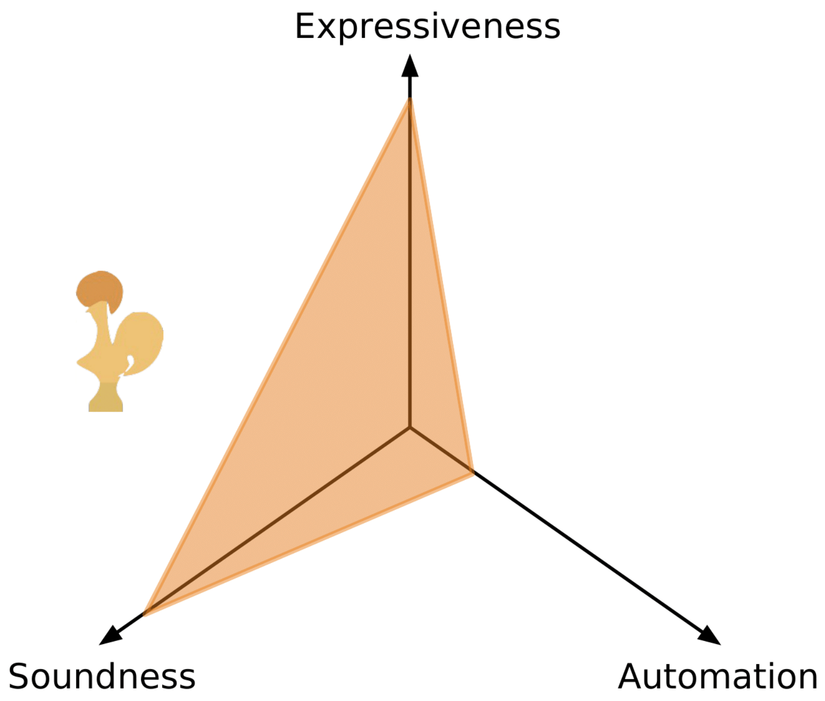
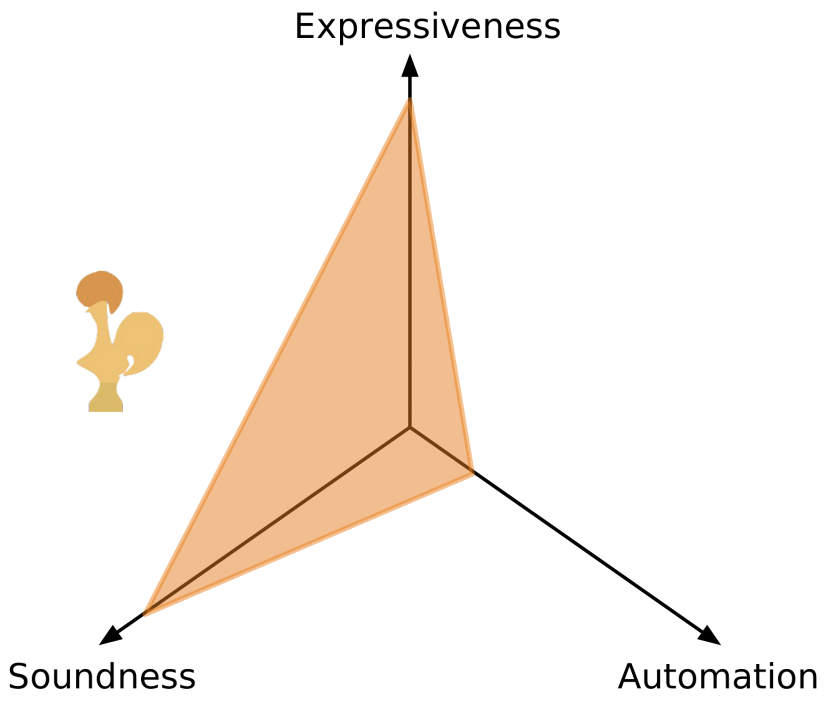
Proof Assistants



Proof Assistants

- Mechanized proofs
- Strong guarantees
- Trusted computing base
- Limited automation



SMT Solvers

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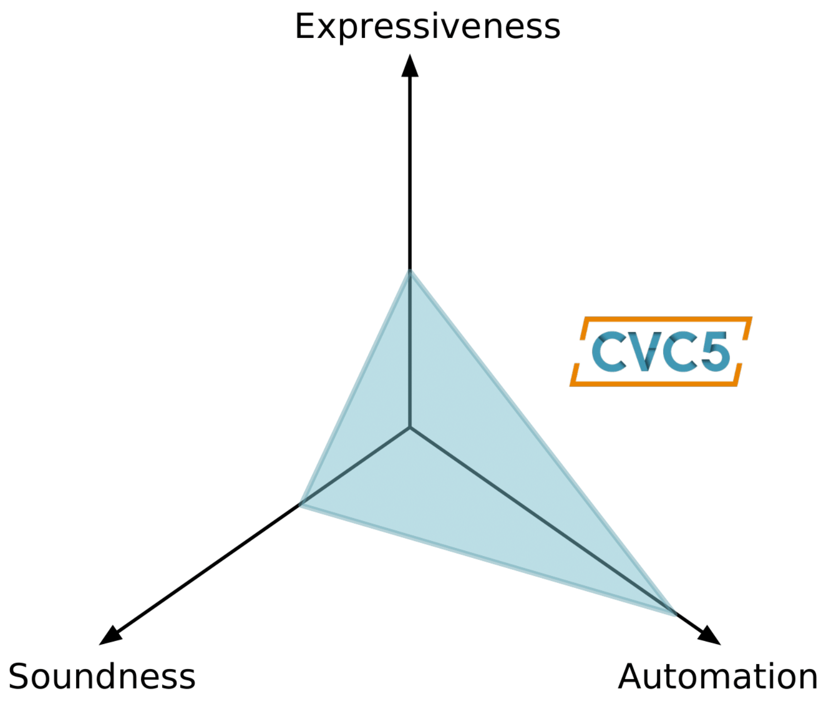

SMT Solvers

- Mechanized proofs
- Strong guarantees
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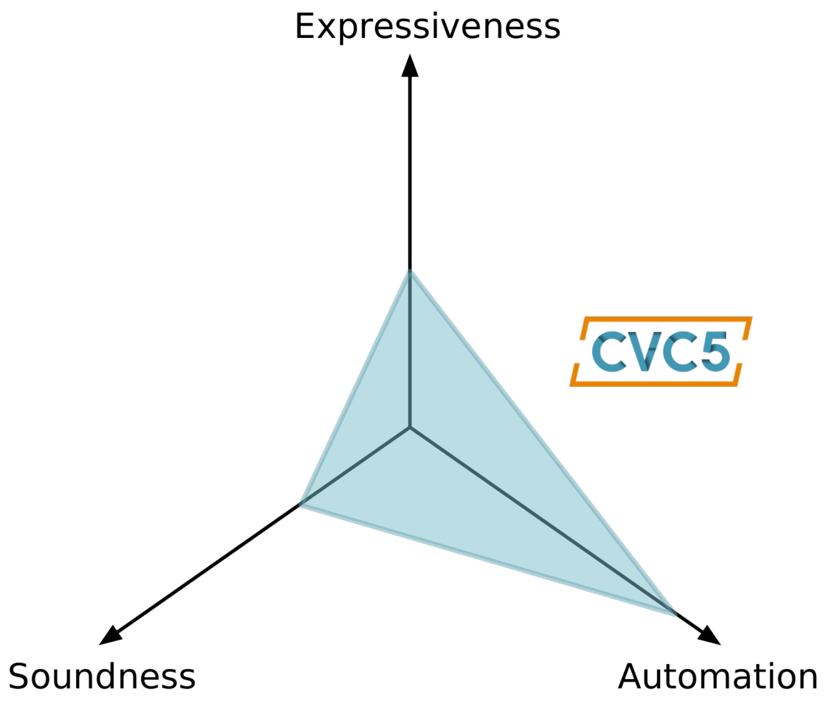
Automated proofs

Vulnerable to bugs

Large code base

High automation



Can we do better?

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- Trusted computing base
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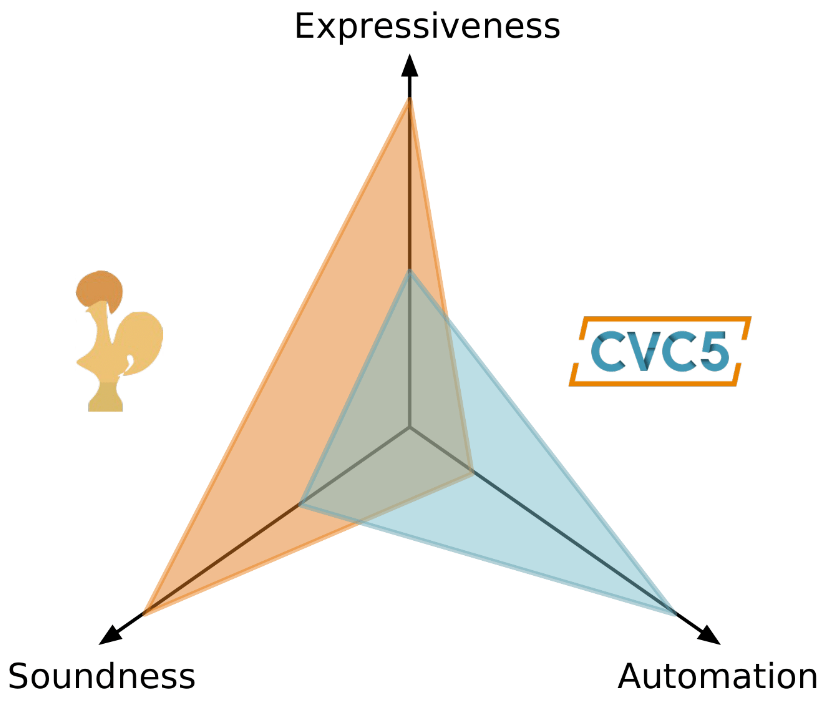
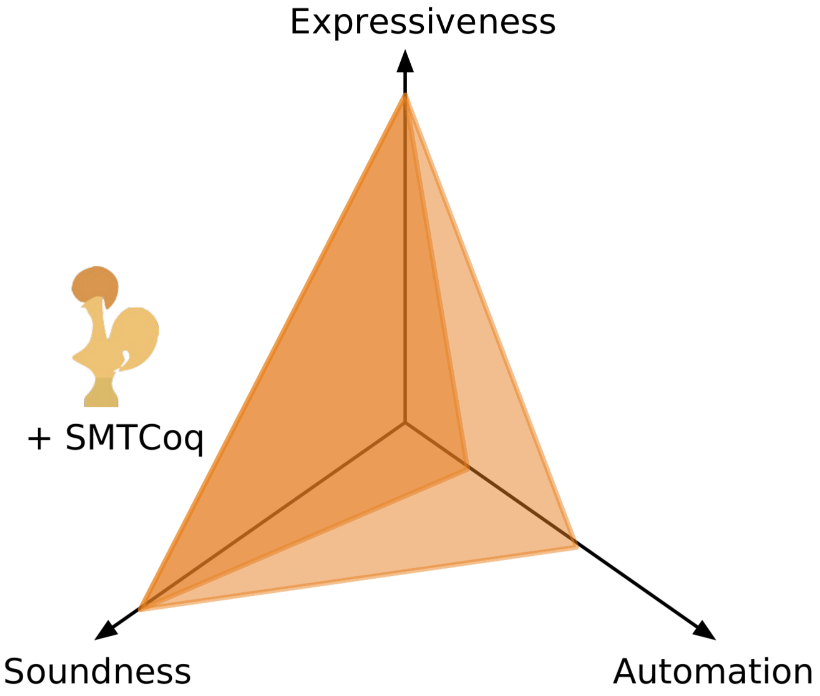
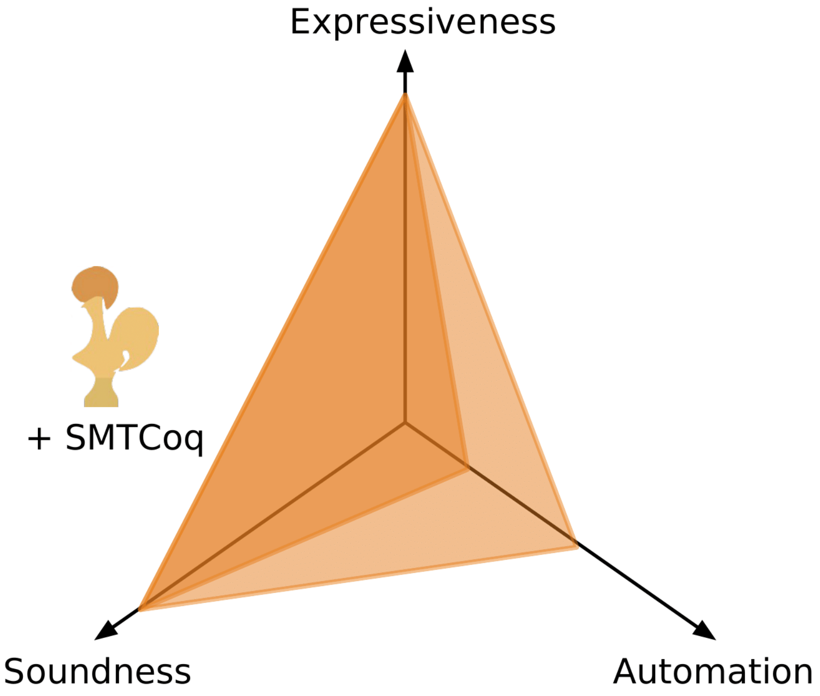


SMTCoq



SMTCoq

- Certified checker
- Automate subgoals
- Uncompromised trusted computing base



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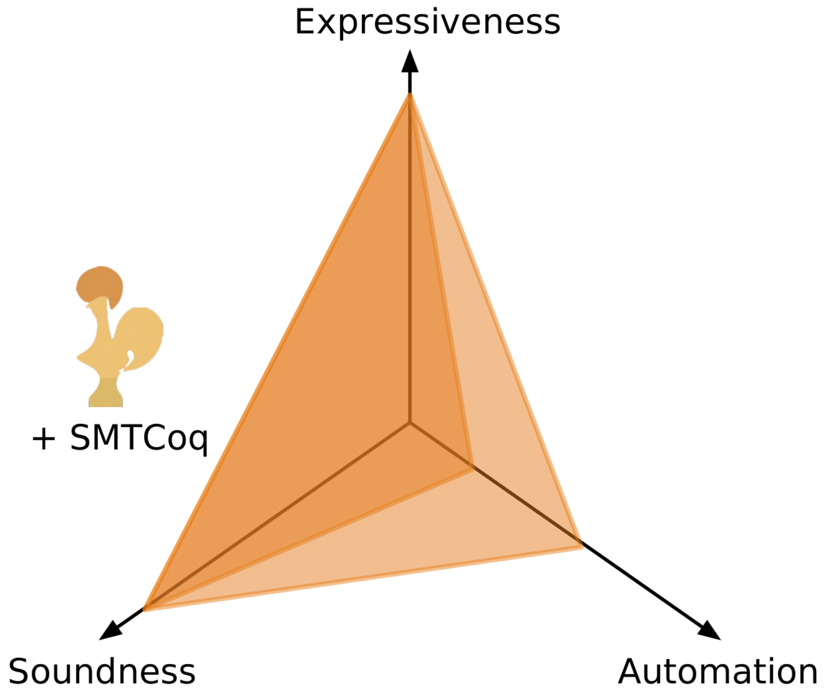
```
Goal forall (x y: Z) (f: Z → Z),
    x = y + 1 → f y = f (x − 1).
Proof.
  intros. rewrite H. rewrite Z.add_simpl_r.
  reflexivity.
Qed.
```



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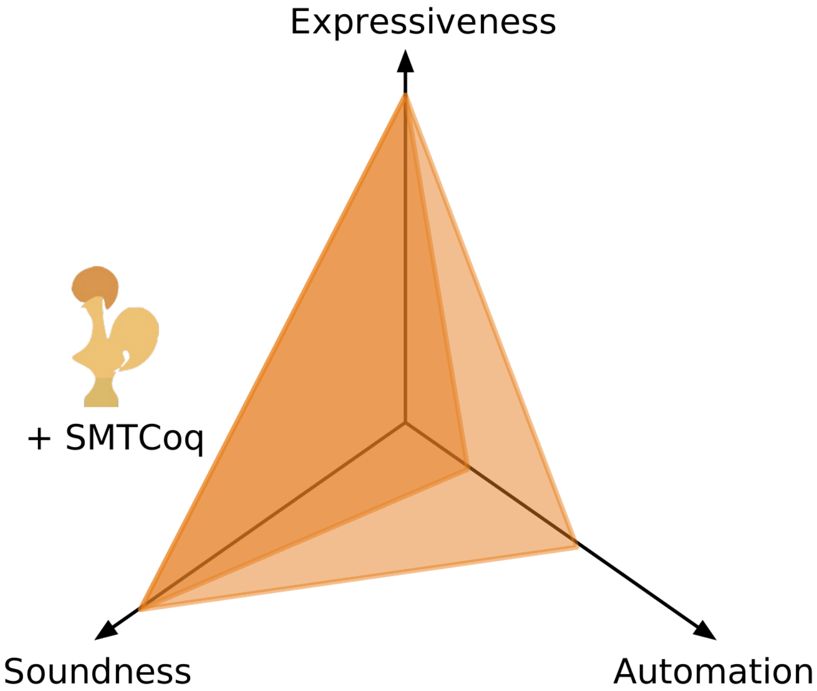

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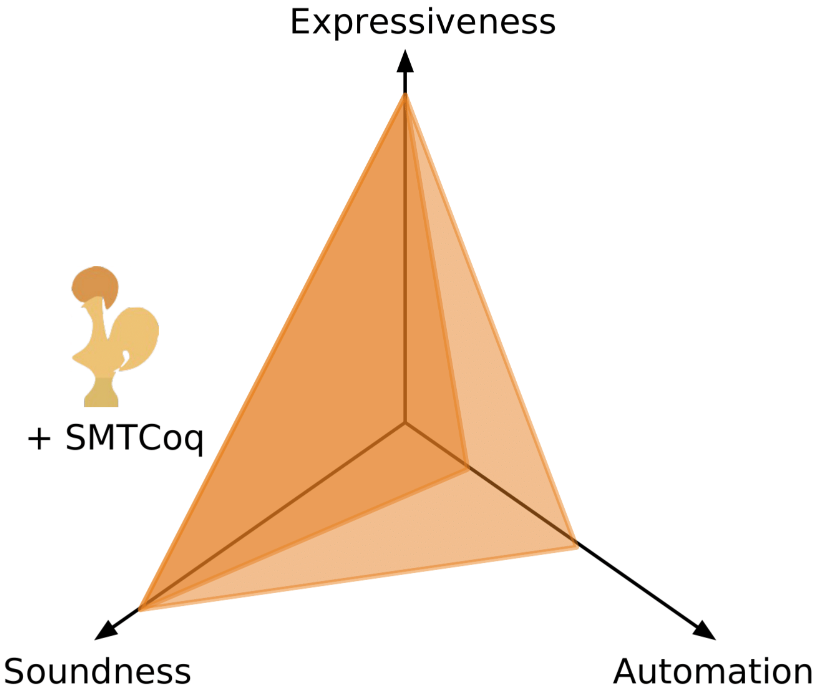
Goal forall (x y: Z) (f: Z → Z),
    f y = f (x − 1).
Proof. smt.
(* Failure! Counter-example:
   x → 0
   y → 1
   f → fun x ⇒ if x = -1 then -2 else 2 *)
```



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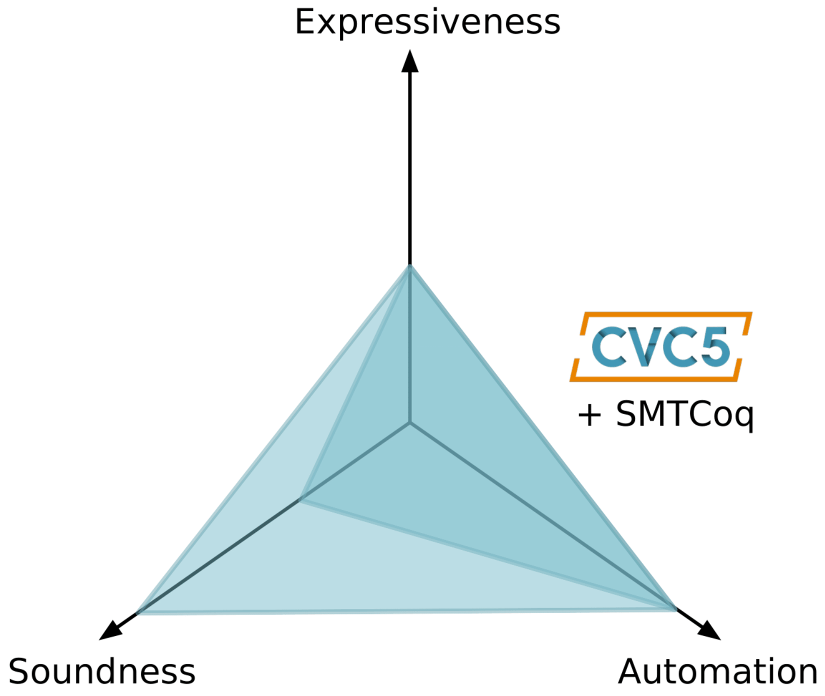
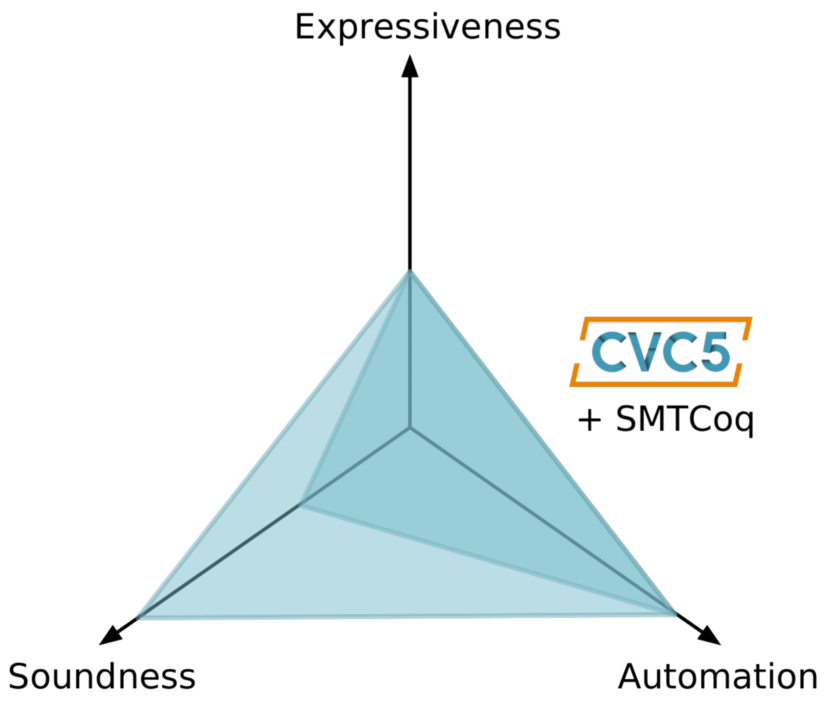


SMTCoq

- Certified checker for SMT Proofs
- Implemented in Coq
- Proven correct in Coq



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```
Goal forall (a b : bool) (x y : Z),
      (ifb a
           (ifb b (2*x + 1 =? 2*y + 1) (2*x + 1 =? 2*y))
           (ifb b (2*x =? 2*y + 1) (2*x =? 2*y)))
      --->
      ((a ---> b) && (b ---> a) && (x =? y)).
Proof. smt. Qed.
```

```
Goal forall (x y : Z), x = y + 1 → x * x = (y + 1) * x.
Proof. smt.
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Proof. smt.
(* Solver error: A non-linear fact was asserted
   to arithmetic in a linear logic. *)
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Definition mul' := Z.mul.
Notation "x *' y" := (mul' x y).
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Proof. smt. Qed.

Goal forall (x y z: Z), x = y + 1 -> y *' z = z *' (x - 1).
Proof. smt.
```

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(* Failure! Counter-example:
   x → 0, y → -1, z → 1,
   mul' → fun x y ⇒ if x = 1 then if y = -1 then -2
                    else 2 else 2 *)
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The `abduce` Tactic

- Present abducts that entail the goal
- Uses abductive reasoning by cvc5

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Goal forall (x y z: Z), x = y + 1 → y *' z = z *' (x − 1).
Proof. (* smt. Failure! *) abduce 3.
(* cvc5 returned SAT.
   The solver cannot prove the goal, but one
   of the following hypotheses would make it provable:
   y = z
   -1 + x = z
   (mul' z y) = (mul' y z) *)
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Goal forall (x y z: Z), x = y + 1 → y *' z = z *' (x - 1).
Proof. (* smt. Failure! abduce 3. *)
  assert ((mul' z y) = (mul' y z)).
  { apply Z.mul_comm. } smt.
Qed.
```

Abduction

- Find A such that
  - $H_1, \ldots, H_n \not\models_T G$
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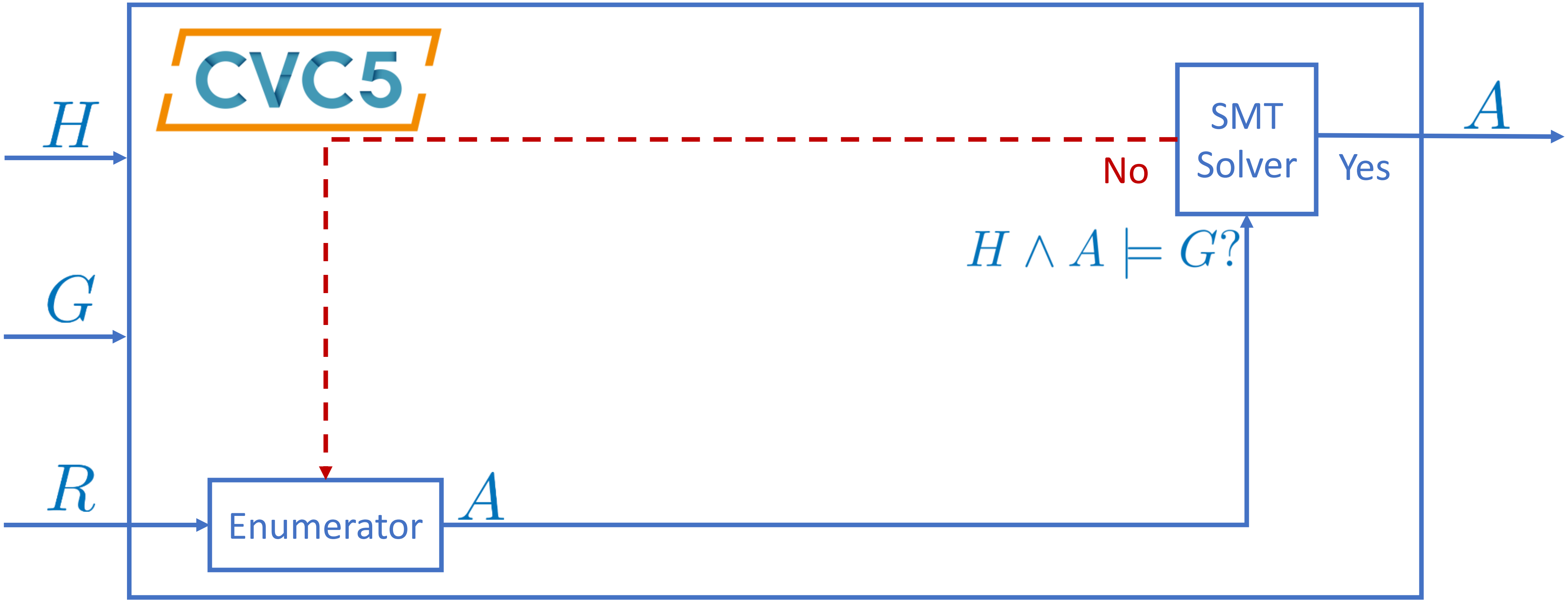
  - A is generated by grammar R

Abduction in cvc5 via SyGuS



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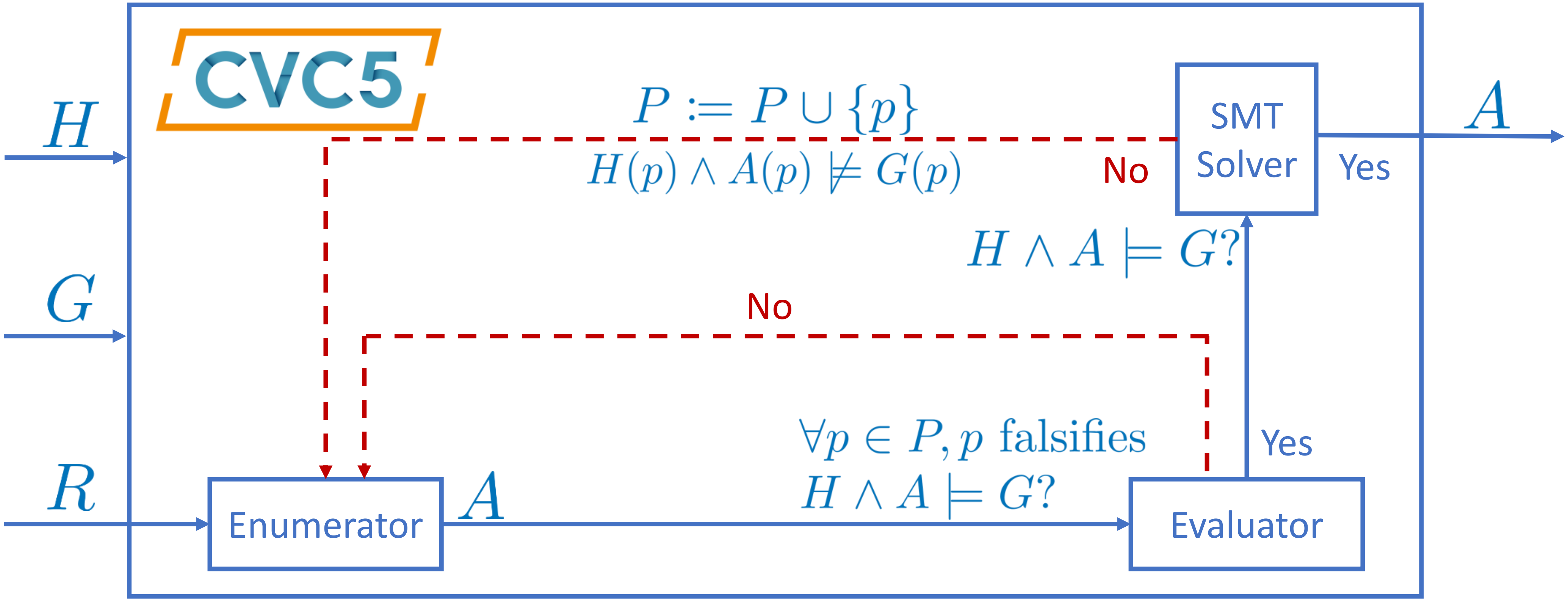

abduce Tactic

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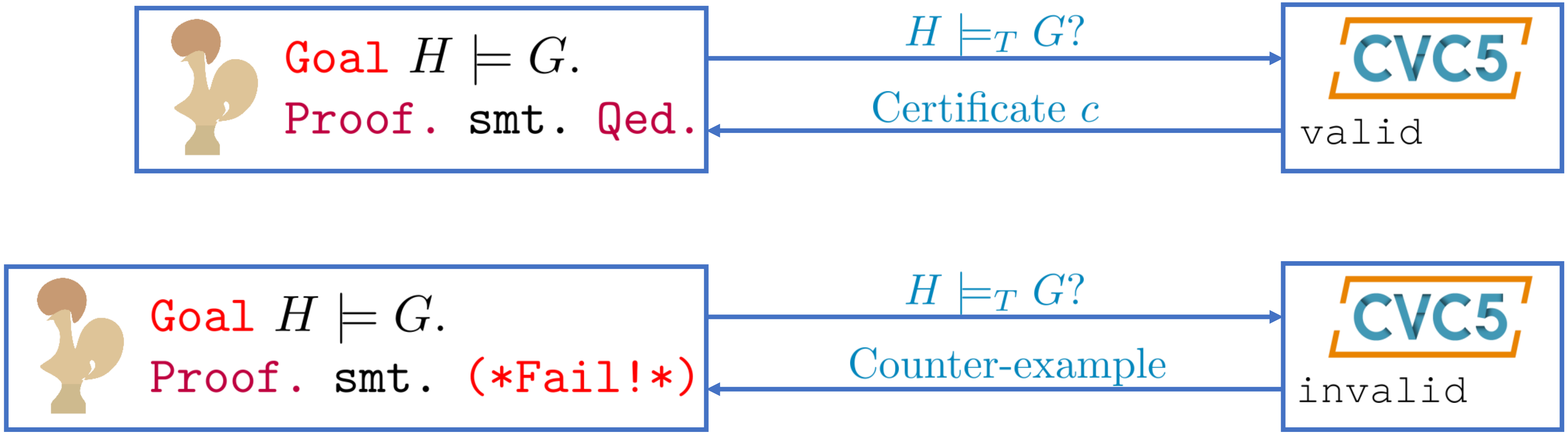
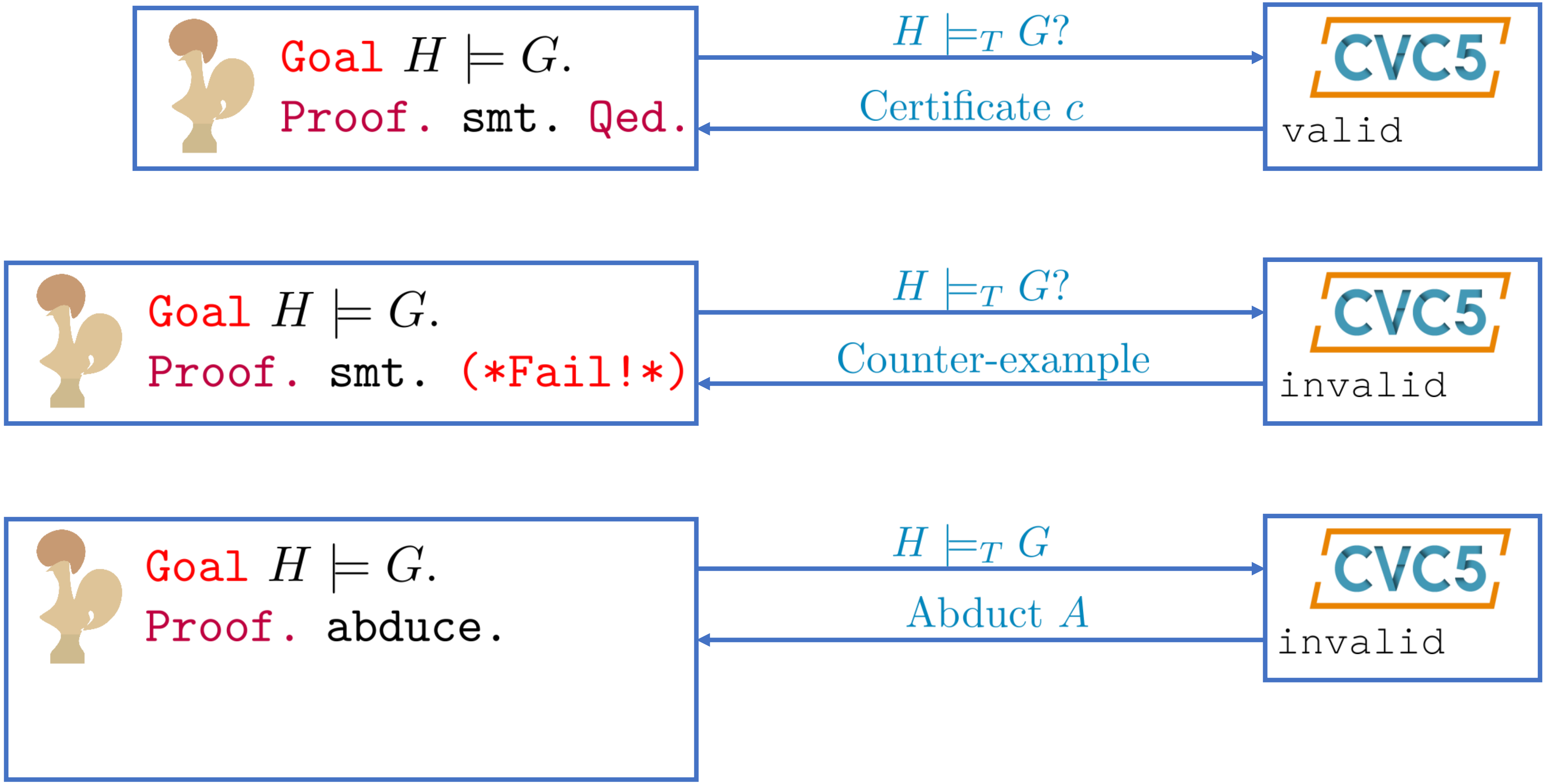
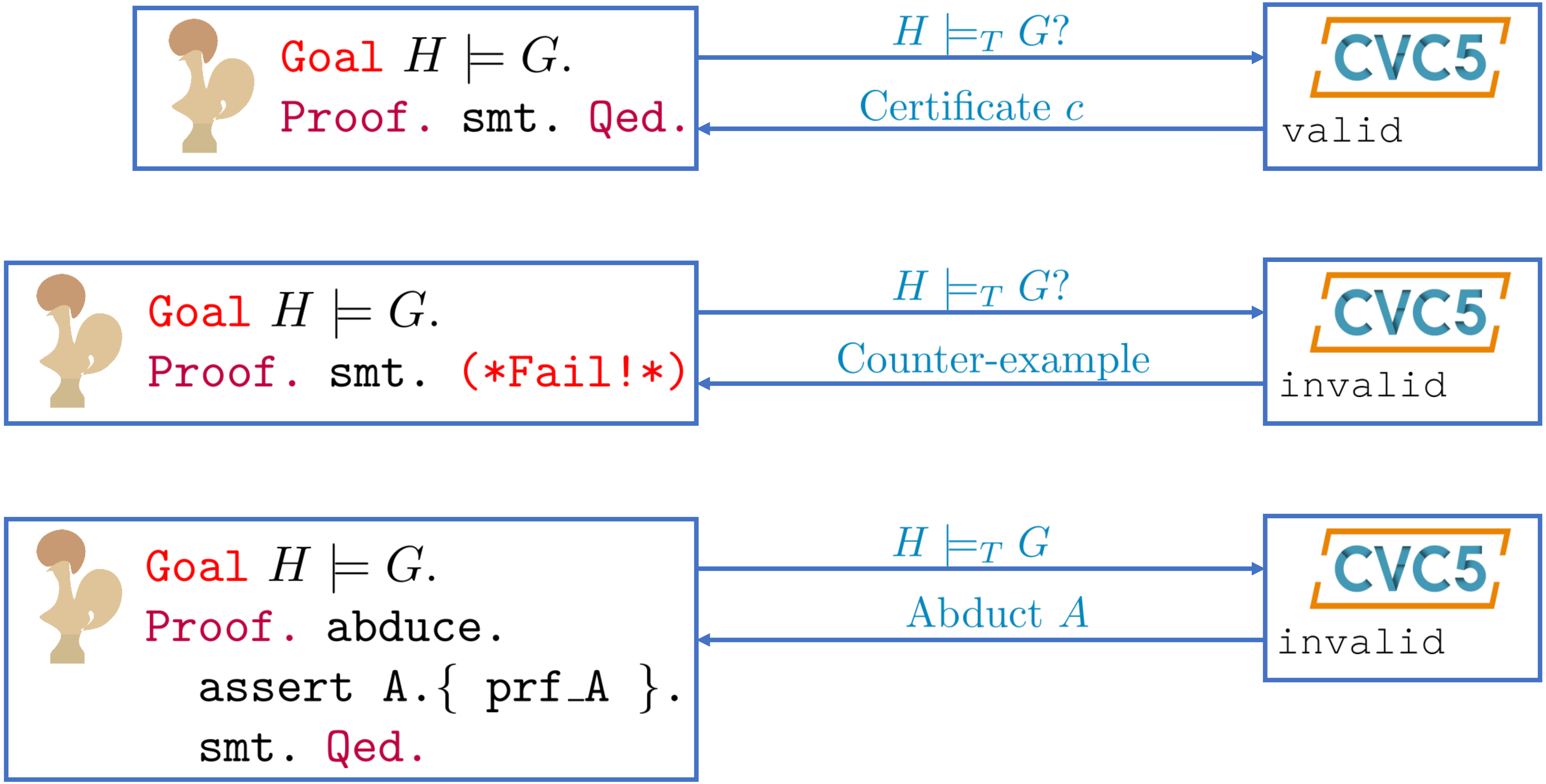


abduce Tactic



abduce Tactic



Evaluation

- On Zorder Coq library

Goals	`smt` Successes	Returns cex	`abduce` Successes	Timeouts
59	33	26	13	13

Evaluation

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- Successor (Z.succ) and predecessor (Z.pred) and functions.

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Lemma Zle_gt_succ n m : n <= m -> Z.succ m > n.
Proof. smt.
```

```
CVC4 returned sat. Here is the model:

n := 0
m := 0
Z.succ := fun _ => 0
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Proof. (* abduce 3. *)
  assert ((Z.succ m) = 1 + m). { unfold Z.succ. smt. }
  smt.
Qed.
```

Future Directions

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- Evaluation inside larger proofs

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- Control abducts by controlling SyGuS grammar

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- Evaluation inside larger proofs
- Control abducts by controlling SyGuS grammar
- Automatically prove entailed abducts

Acknowledgements

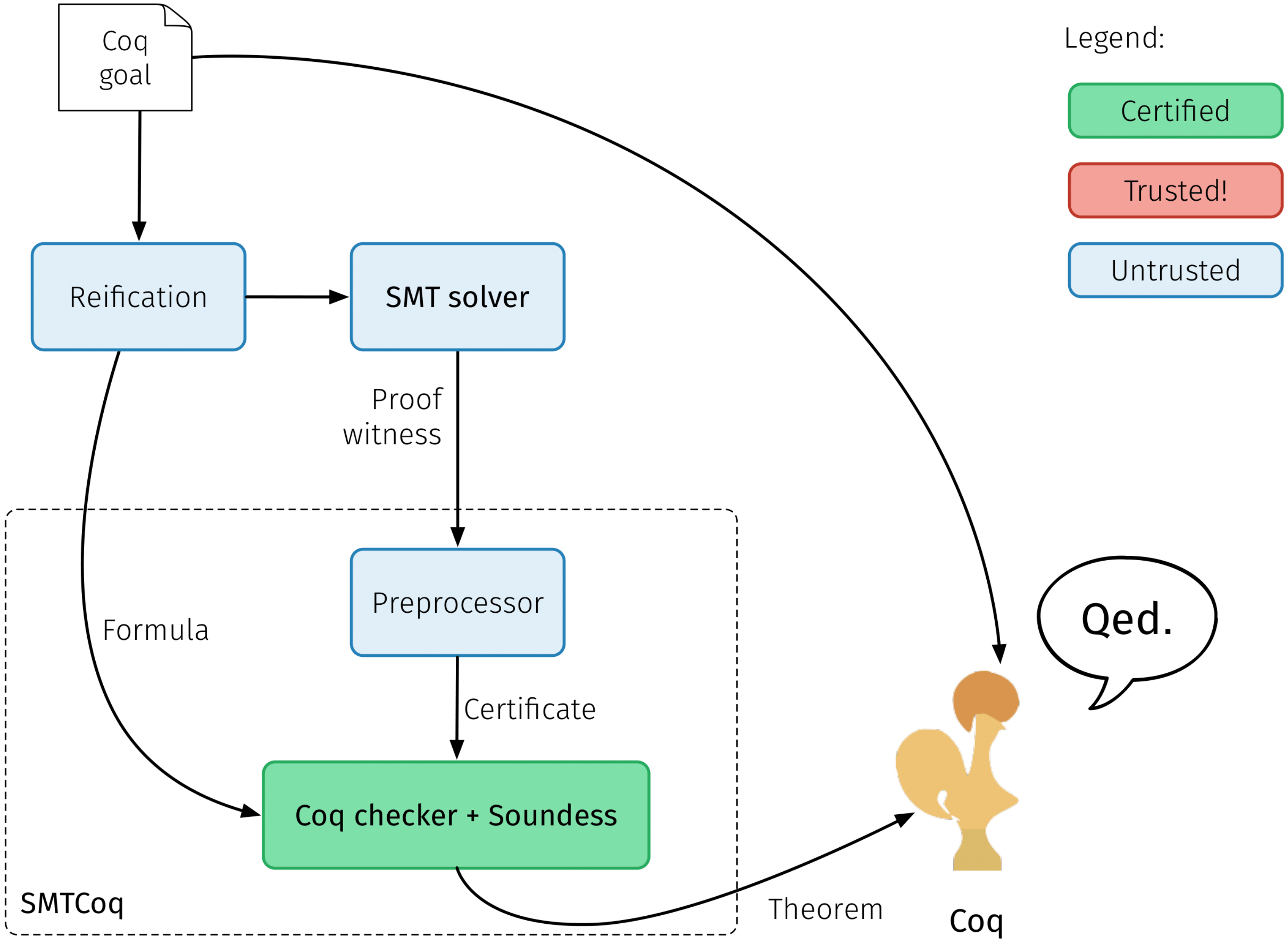
- https://smtcoq.github.io/

- Scalable Algorithms for Abduction via Enumerative Syntax-Guided Synthesis. *IJCAR 2020. Andrew Reynolds, Haniel Barbosa, Daniel Larraz, and Cesare Tinelli*

- Images borrowed from slides presented by Alain Mebsout at CAV 2017 – “SMTCoq: A plug-in for integrating SMT solvers into Coq"

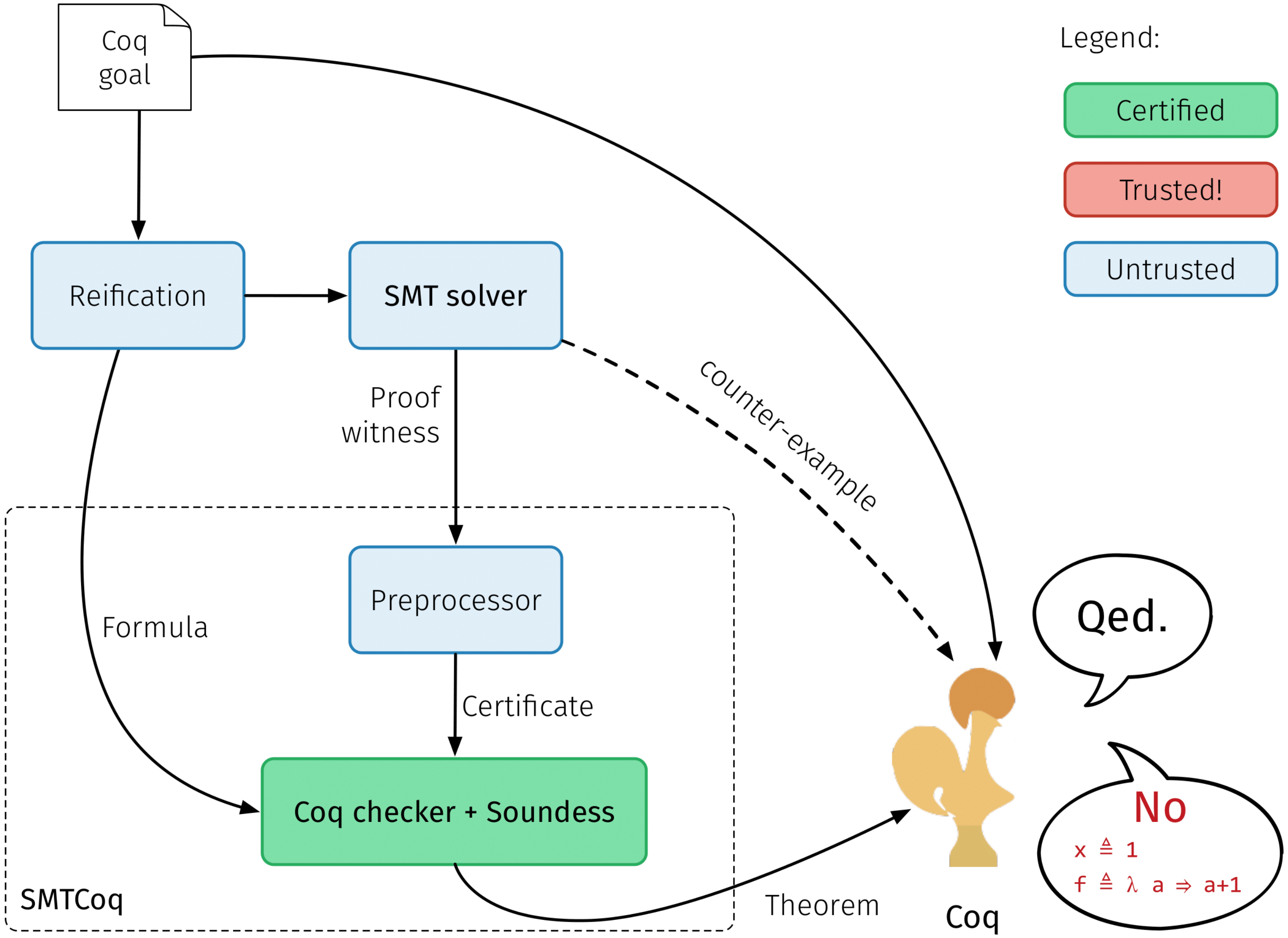
Backup Slides

SMTCoq



Coq
goal
Legend:
Certified
Trusted!
Untrusted
Reification
SMT solver
Proof
witness
Preprocessor
Formula
Certificate
Coq checker + Soundess
SMTCoq
Theorem
Qed.
Coq

SMTCoq



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x ≜ 1
f ≜ λ a ⇒ a+1