CS:4420 Artificial Intelligence

Spring 2019

Uninformed Search

Cesare Tinelli

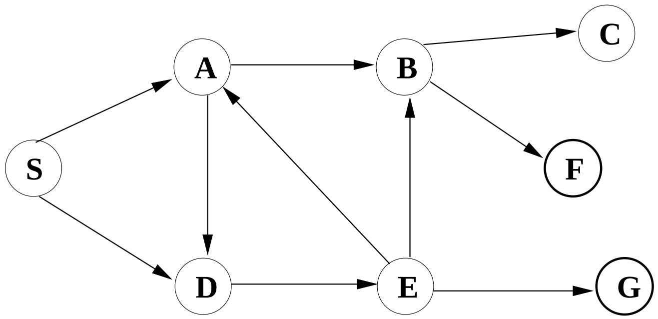
The University of Iowa

Readings

- Chap. 3 of [Russell and Norvig, 3rd edition]

More on Graphs

A *graph* is a set of *nodes* and *edges* between them

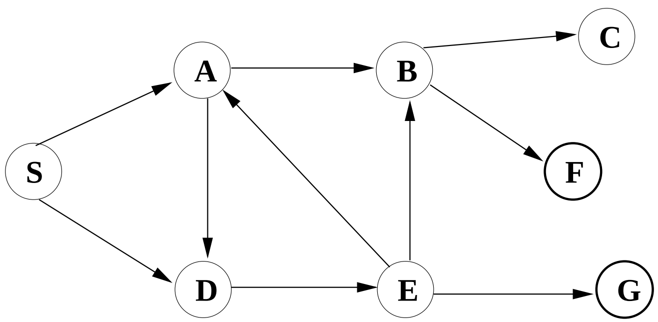


A graph is *directed* if its edges can be traversed only in a specified direction

When an edge is directed from n_i to n_j

- it is uniquely identified by the pair (n_i, n_j)
- n_i is a parent (or *predecessor*) of n_j
- n_j is a *child* (or *successor*) of n_i

Directed Graphs



A *path*, of length $k \geq 0$, is a sequence $\langle (n_1, n_2), (n_2, n_3), \ldots, (n_k, n_{k+1}) \rangle$ of k *successive* edges [a]

Ex: $\langle \rangle$, $\langle (S, D) \rangle$, $\langle (S, D), (D, E), (E, B) \rangle$

For $1 \leq i < j \leq k + 1$,

- N_i is a *ancestor* of N_j; N_j is a *descendant* of N_i

A graph is *cyclic* if it has a path starting and ending with the same node. *Ex:* $\langle (A, D), (D, E), (E, A) \rangle$

[a] Note that a path of length $k > 0$ contains $k + 1$ nodes

From Search Graphs to Search Trees

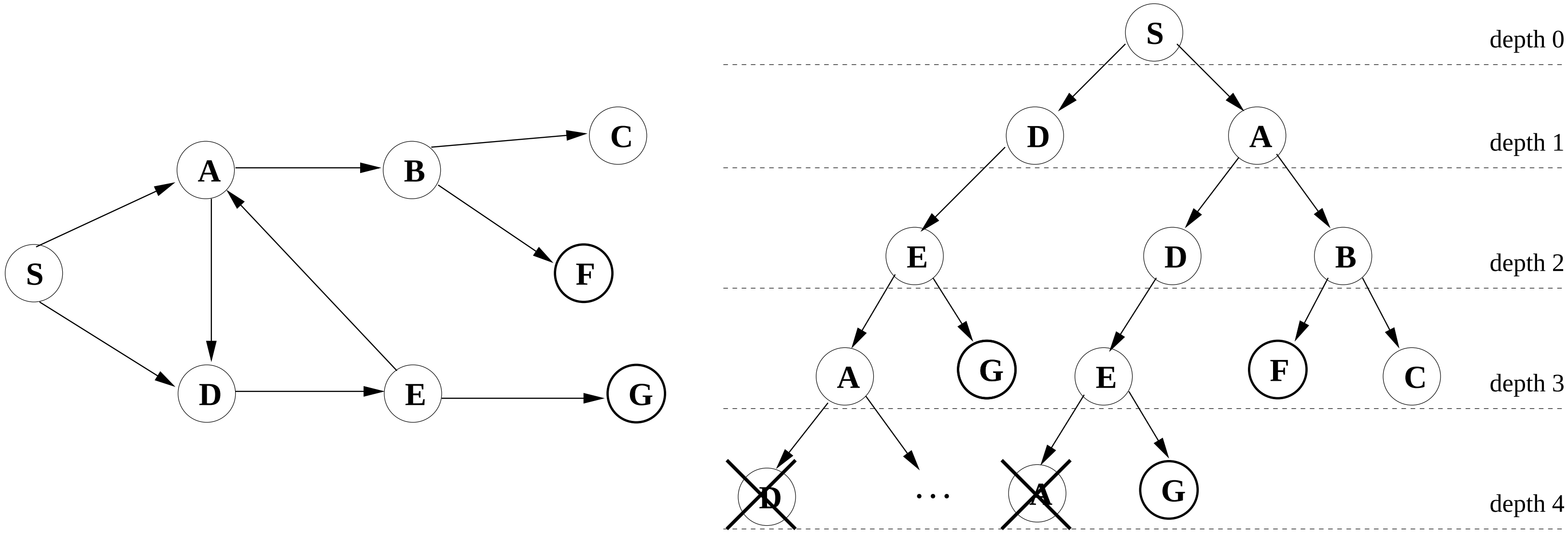
The set of all possible paths of a graph can be represented as a tree

- A *tree* is a directed acyclic graph all of whose nodes have at most one parent
- A *root* of a tree is a node with no parents
- A *leaf* is a node with no children
- The *branching factor* of a node is the number of its children

Graphs can be turned into trees by duplicating nodes and breaking cyclic paths, if any

From Graphs to Trees

To unravel a graph into a tree choose a root node and trace every path from that node until you reach a leaf node or a node already in that path



Note:

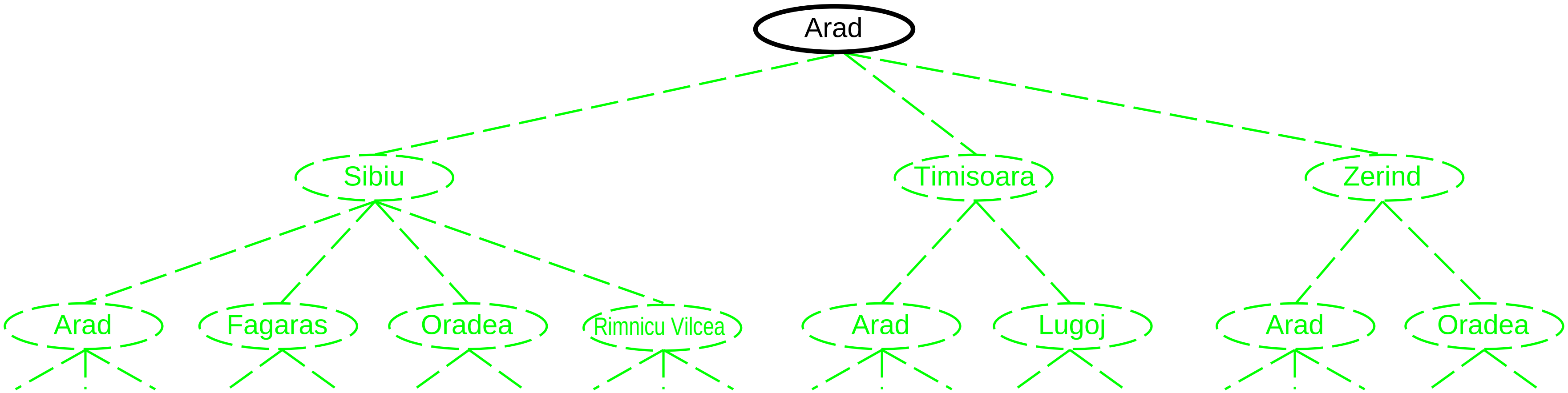
- must remember which nodes have been visited
- a node may get duplicated several times in the tree
- the tree has infinite paths if and only if the graph has infinite non-cyclic paths

Tree Search Algorithms

Basic Idea: offline, simulated exploration of state space by generating successors of already-explored states

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then
            return failure
        else
            choose a leaf node for expansion according to strategy
            if the node contains a goal state then
                return the corresponding solution
            else
                expand the node and add its successors to the tree
    done
```

Tree Search Example



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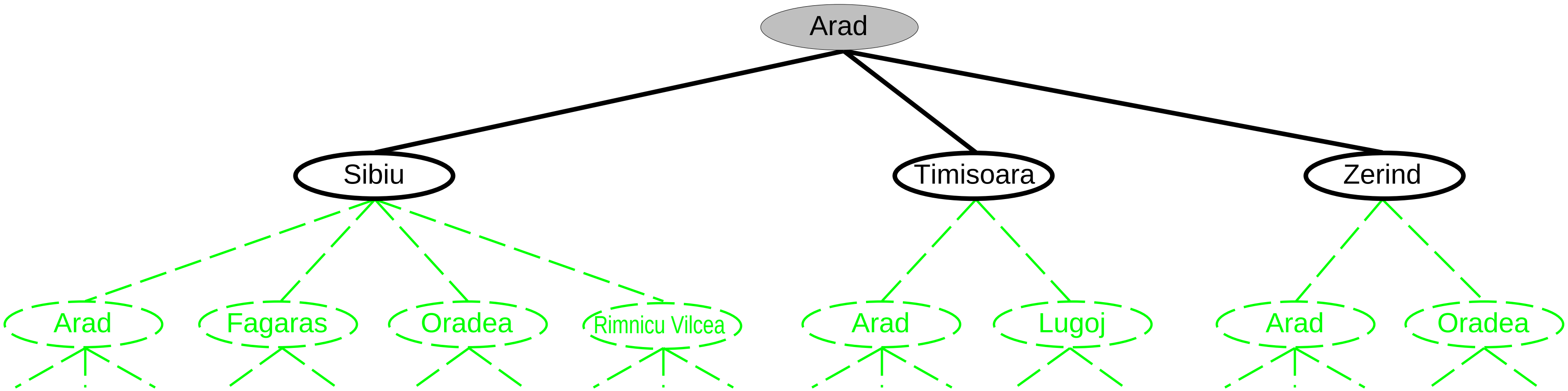

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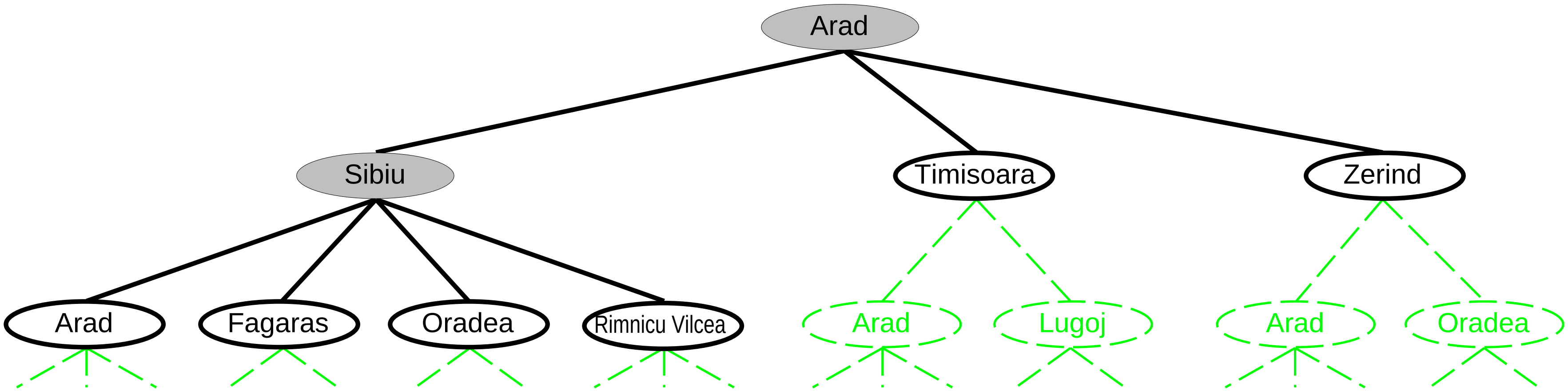
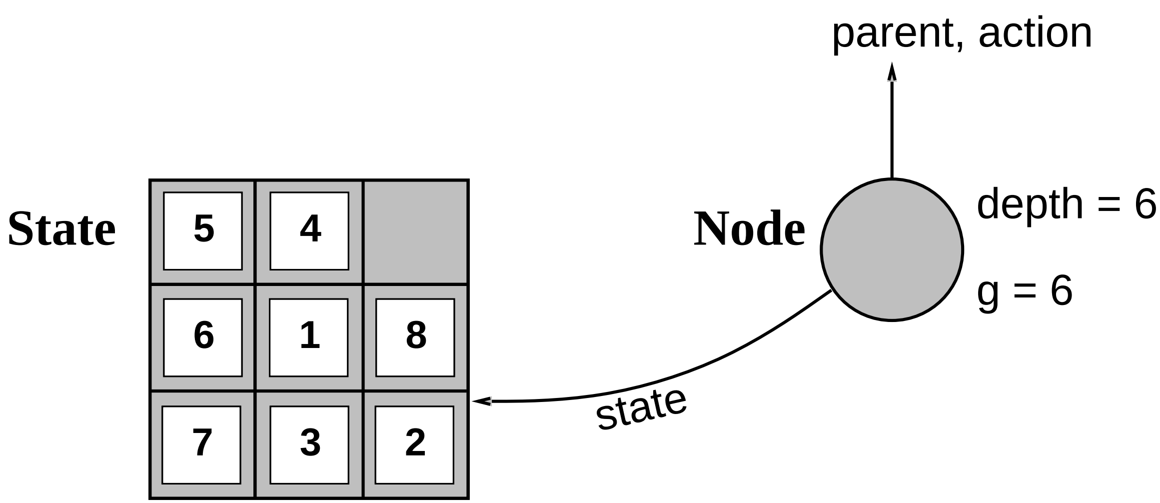


Implementation: states vs. nodes

- A *state* is a (representation of) a physical configuration
- A *node* is a data structure constituting part of a search tree (and includes such info as parent, children, depth, *path cost* $g(x)$)
- States do not have parents, children, depth, or path cost!



Search Strategies

A strategy is defined by picking the order of node expansion

Strategies are evaluated along the following dimensions:

- *solution completeness*: does it always find a solution if one exists?
- *time complexity*: number of nodes generated/expanded
- *space complexity*: maximum number of nodes in memory
- *optimality*: does it always find a least-cost solution?

Time and space complexity are measured in terms of

- b, maximum branching factor of the search tree
- d, depth of the least-cost solution
- m, maximum depth of the state space (may be ∞)

Search Strategies

Uninformed (or Blind) Search Strategies

- Little or no information about the search space is available
- All we know is how to generate new states and recognize a goal state

Informed (or Heuristic) Search Strategies

- An estimate of the number of steps or the path cost from current state to goal state is available
- The estimate is not perfect (otherwise no search is needed!) but can help prune the search space considerably

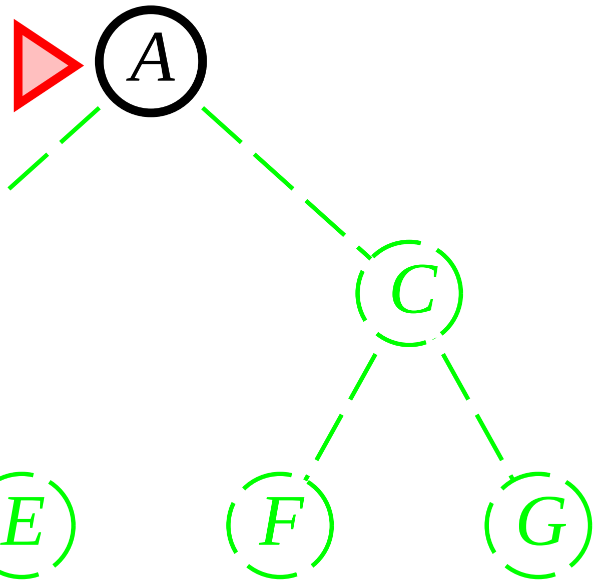
Some Uninformed Search Strategies

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening (depth-first) search

Breadth-First Search

Strategy: Expand shallowest unexpanded node

Implementation: the current set of unexpanded nodes, the *fringe (or frontier)*, is processed as FIFO queue, i.e., new successors go at end



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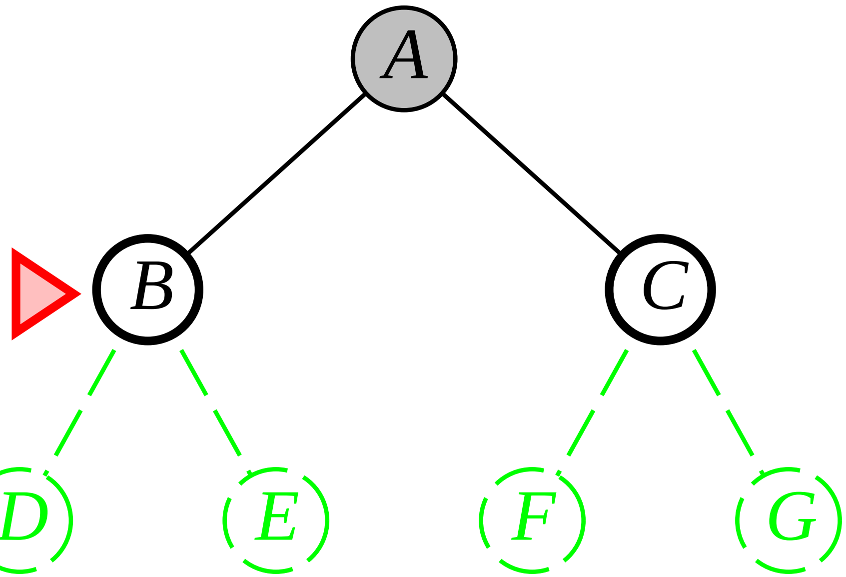

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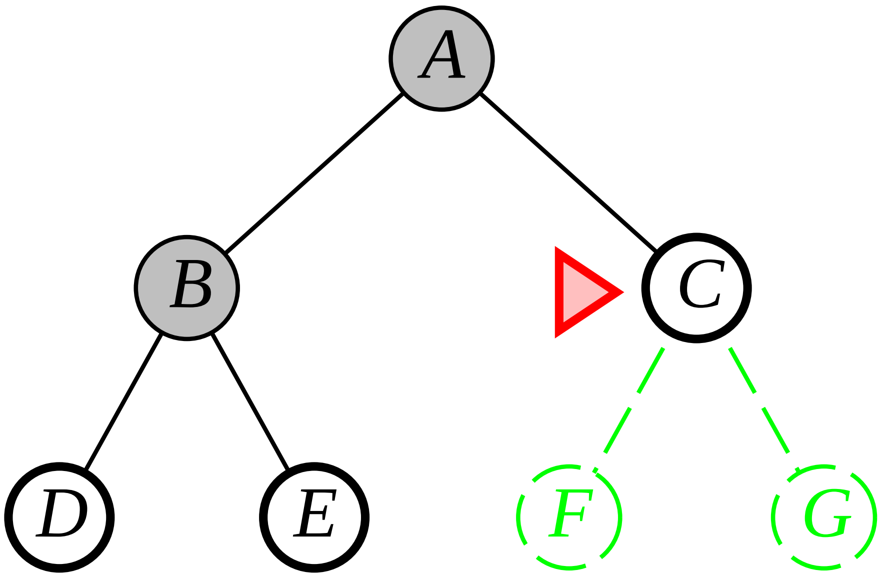

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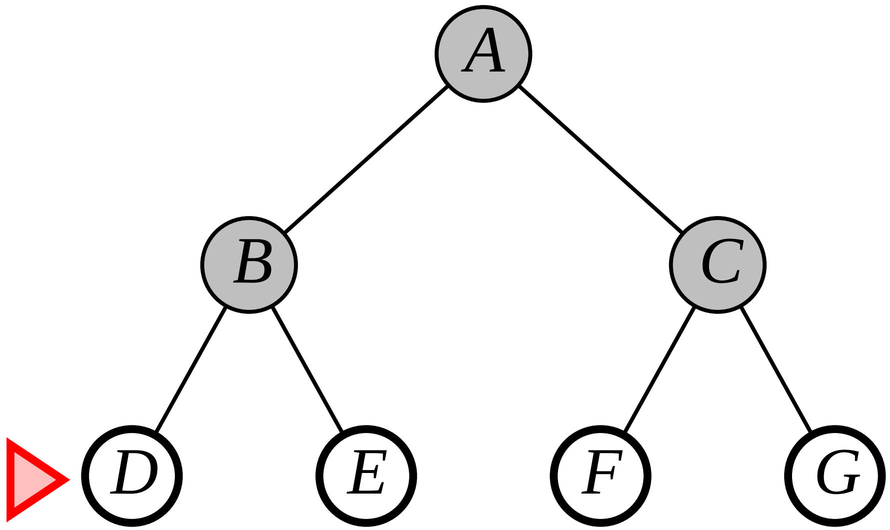
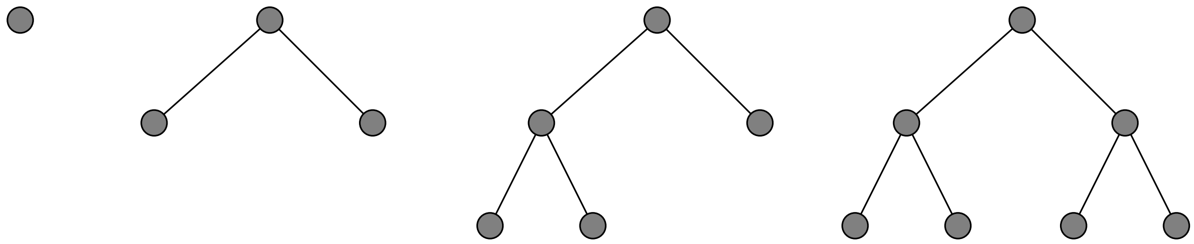


Cost of Breadth-First Search



Worst-case Time Complexity (no. of node expansions)

All nodes must be expanded to find a goal state. We must process these many nodes:

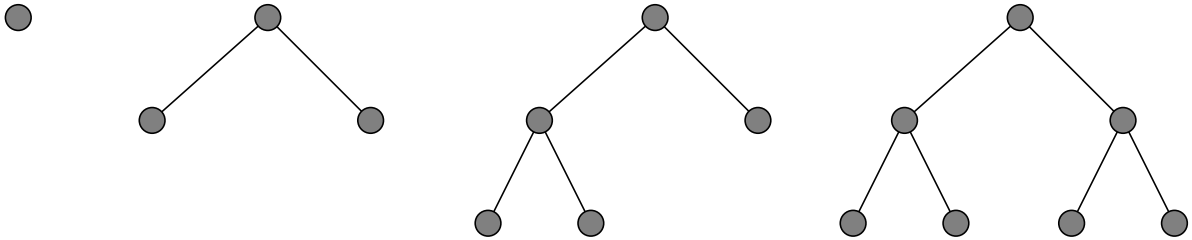
$O(1 + b + b^2 + \ldots + b^d + b(b^d - 1)) = O(b^{d+1})$ (exponential time)

where $b =$ maximum branching factor

$d =$ depth of shallowest goal state

Note: The above assumes that the search space if finite. What if it is not?

Cost of Breadth-First Search



Worst-case Space Complexity (no. of nodes in memory)

All nodes at depth d of the search tree are in the fringe when the procedure finds the goal state

The number of nodes at depth d in a tree with branching factor b is

$$O(b^{d+1}) \qquad \text{(exponential space)}$$

Cost of Breadth-First Search

Depth	Nodes	Time	Memory
0	1	1 millisecond	100 bytes
2	111	.1 seconds	11 kilobytes
4	11,111	11 seconds	1 megabyte
6	10^6	18 minutes	111 megabytes
8	10^8	31 hours	11 gigabytes
10	10^{10}	128 days	1 terabyte
12	10^{12}	35 years	111 terabytes
14	10^{14}	3500 years	11,111 terabytes

$b = 10$, time/node=1ms, mem/node= 100bytes

- Exponential complexity problems become soon unmanageable
- Memory requirements are a bigger problem than time requirements

Optimality of Breadth-First Search

Breadth-first search is clearly complete.

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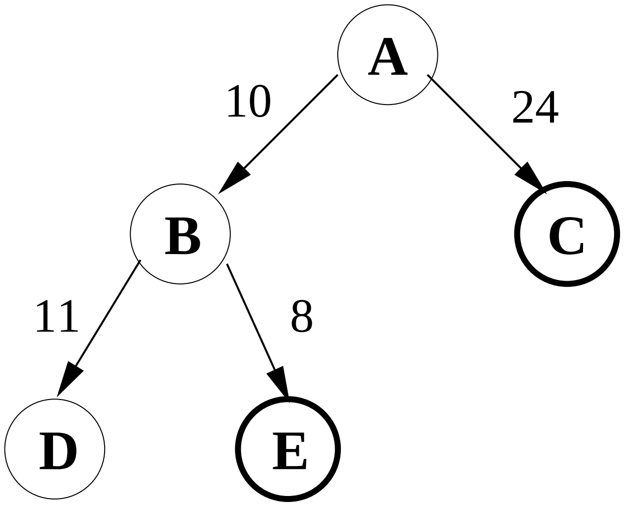
- Breadth-first search always finds the shallowest goal state
- The path to that goal state, however, may have a higher cost than one to a deeper goal state

Optimality of Breadth-First Search

Breadth-first search is clearly complete. Is it optimal? It depends

- Breadth-first search always finds the shallowest goal state
- The path to that goal state, however, may have a higher cost than one to a deeper goal state



Cost of $\mathbf{AC}$: 24

Cost of $\mathbf{ABE}$: $10 + 8 = 18$

If we are looking for least-cost solutions, breadth-first is suboptimal unless all step costs are identical

Uniform-Cost Search

Assumption: A path cost function g such that $g(p) - g(p') \geq \epsilon > 0$ for all paths p and proper subpaths p' of p

Strategy: Expand least-cost unexpanded node

Implementation: $fringe =$ priority queue ordered by path cost

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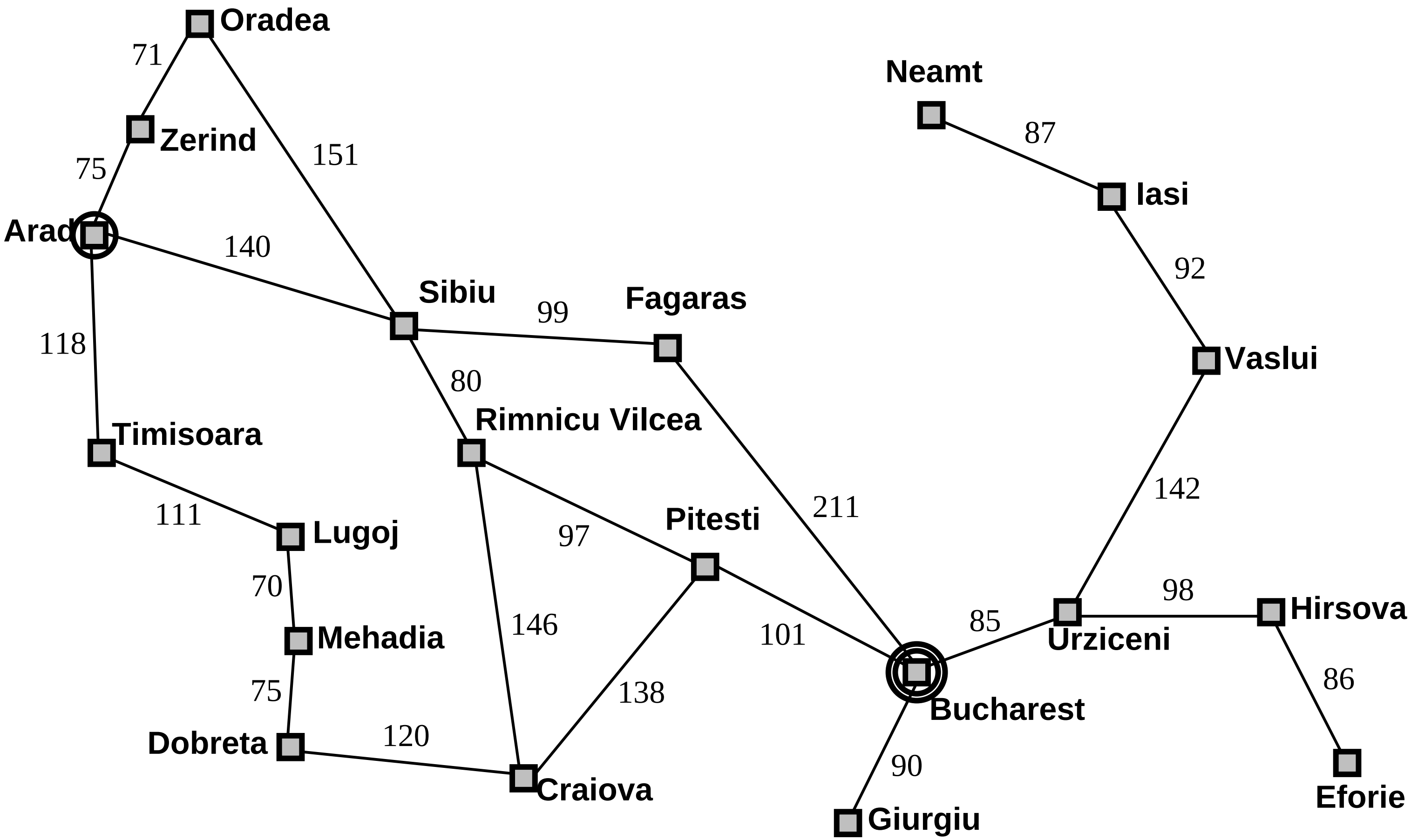
Equivalent to breadth-first if step costs all equal

Uniform-Cost Search: Example

Path cost = sum of step costs



Exercise: Find cheapest route from Sibiu to Bucharest

Properties of Uniform-Cost Search

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Implementation: $fringe =$ priority queue ordered by path cost

Complete?

Time complexity?

Space complexity?

Optimal?

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Complete? Yes (with step cost $\geq \epsilon$)

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Complete? Yes (with step cost $\geq \epsilon$)

Time complexity? $\#$ of paths p with $g(p) \leq$ cost of optimal solution: $O(b^{\lceil C^*/\epsilon \rceil})$ where C^* is the cost of the optimal solution

Space complexity?

Optimal?

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Optimal?

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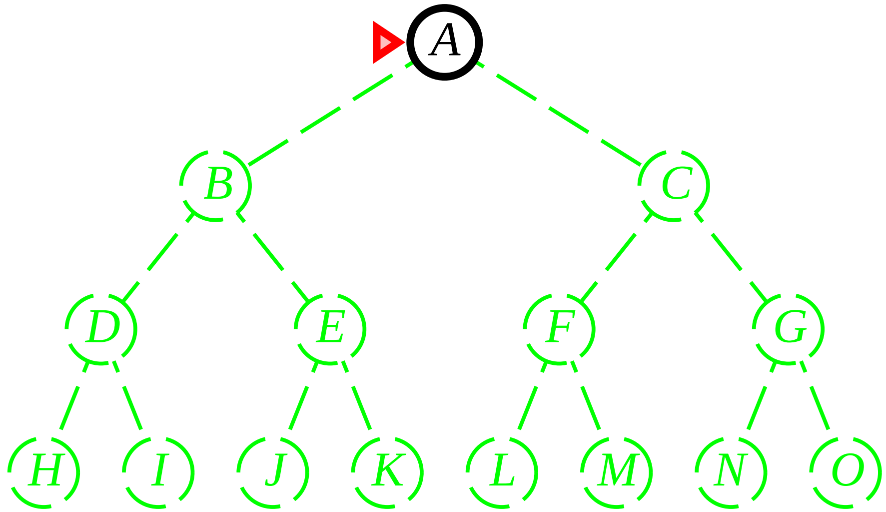
Space complexity? Same as time complexity: $O(b^{\lceil C^*/\epsilon \rceil})$

Optimal? Yes, since nodes are expanded in increasing order of g

Depth-First Search

Strategy: Expand deepest unexpanded node

Implementation: $fringe =$ LIFO queue, i.e., put successors at front



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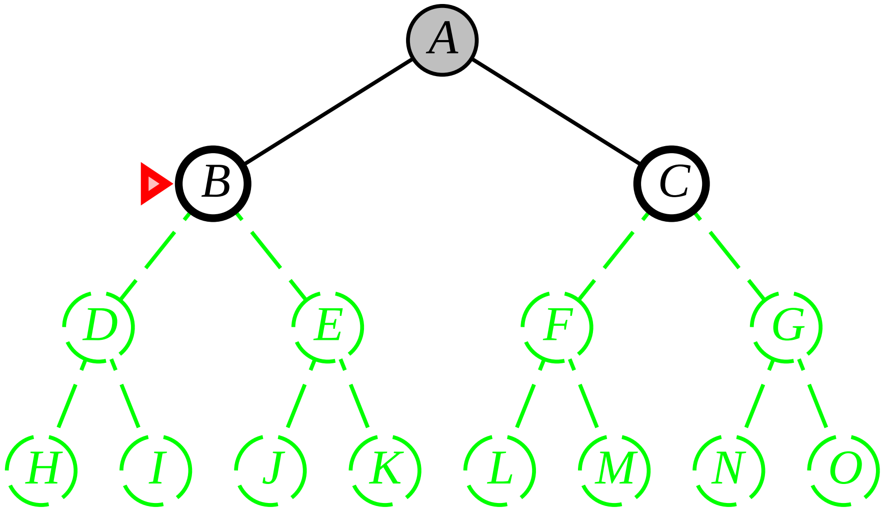

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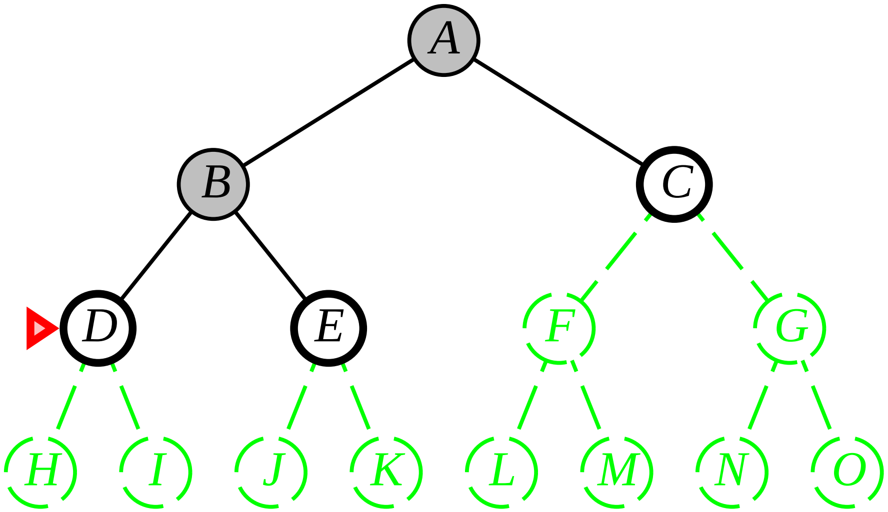

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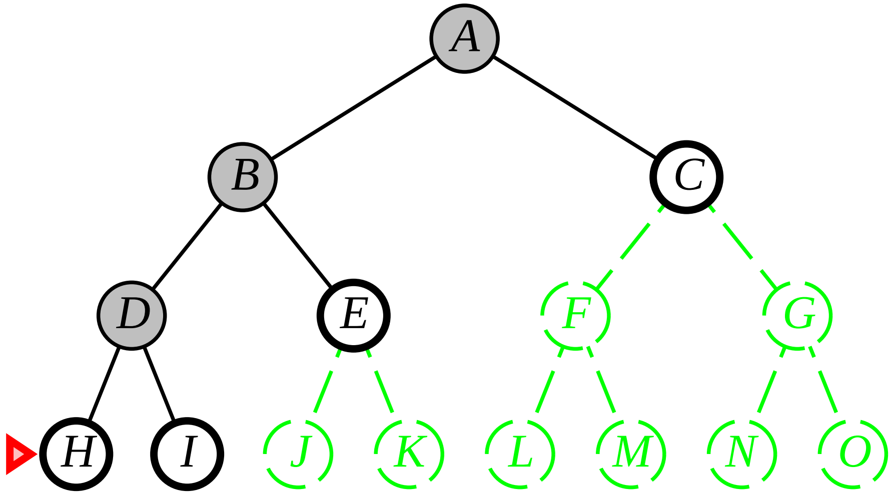

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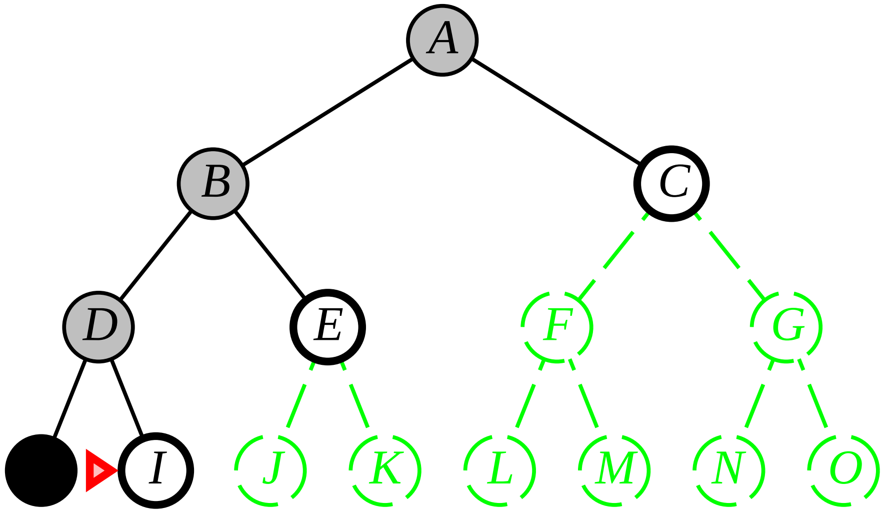

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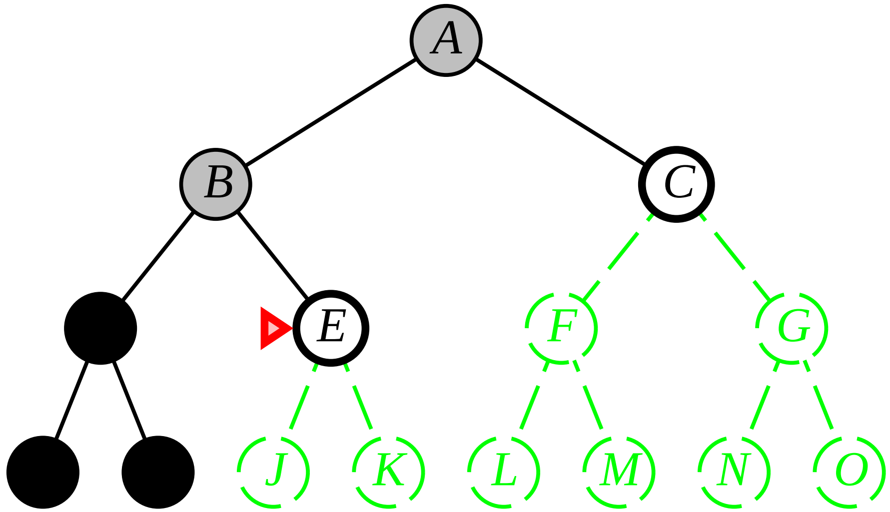

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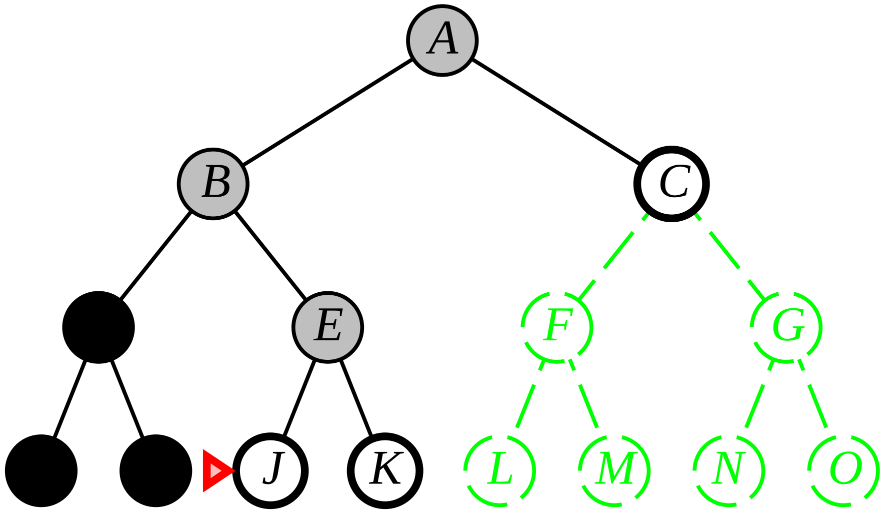

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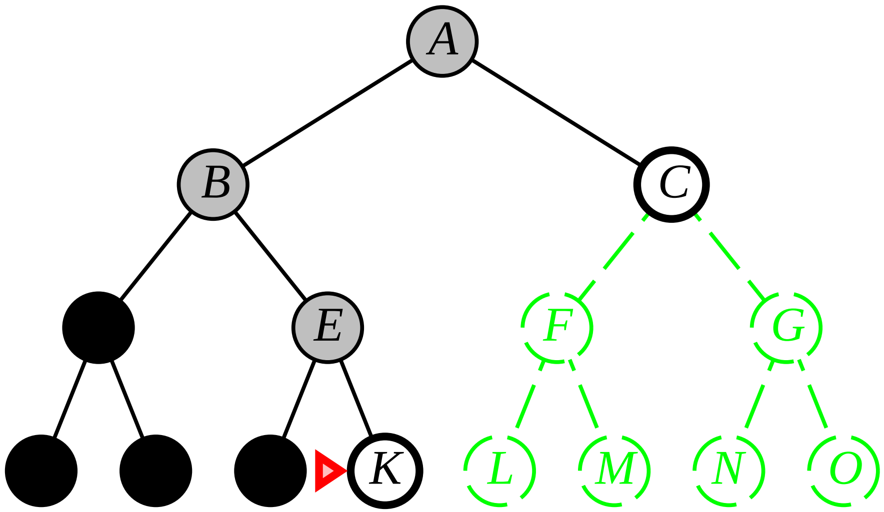

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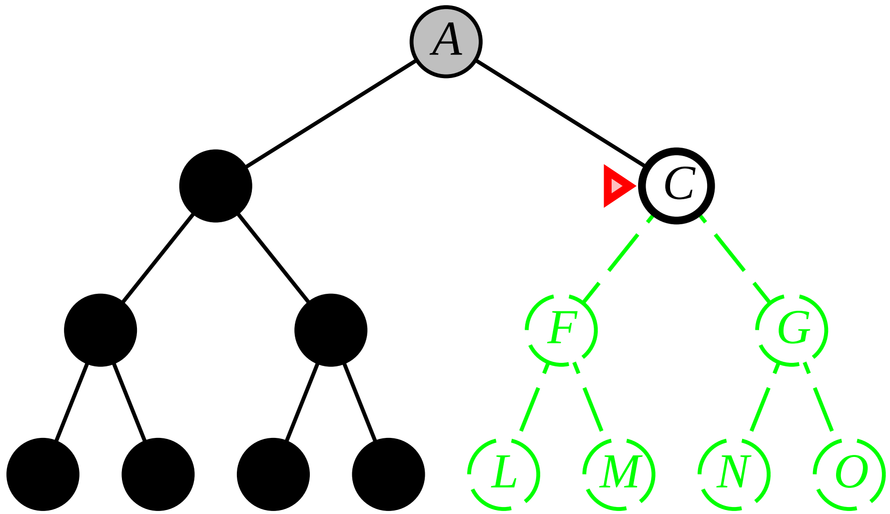

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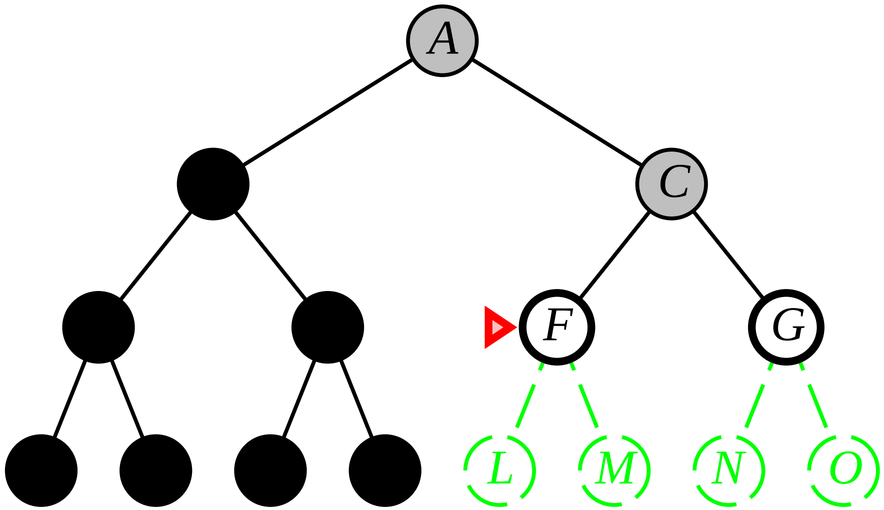

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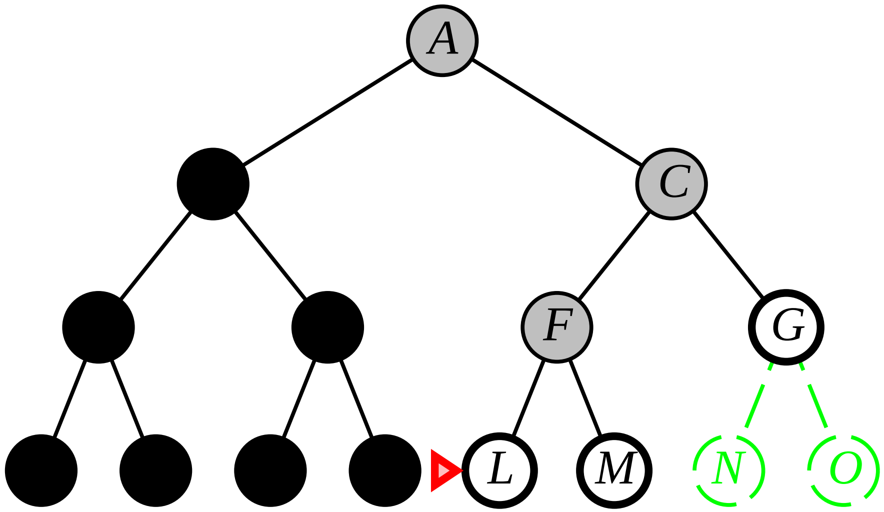

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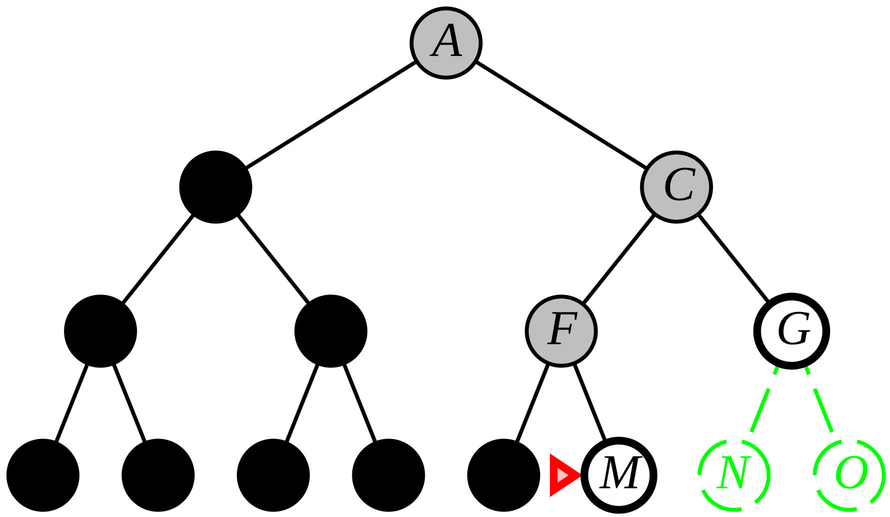

Properties of Depth-First Search

Complete?

Time complexity?

Space complexity?

Optimal?

Properties of Depth-First Search

Complete? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path $\Rightarrow$ complete in finite spaces

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Space complexity? $O(bm)$, i.e., linear space!

Optimal? No

Depth-Limited Search

= depth-first search with depth limit l, i.e., nodes at depth l have no successors

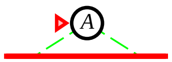
```
function Depth-Limited-Search (problem, limit) return soln/fail/cutoff
   return Recursive-DLS(Make-Node(Initial-State(problem)), problem, limit)
end function

function Recursive-DLS (node, problem, limit) return soln/fail/cutoff
   cutoff-occurred := false;
   if (Goal-State(problem, State(node))) then return node;
   else if (Depth(node) == limit) then return cutoff;
   else for each successor in Expand(node, problem) do
           result := Recursive-DLS(successor, problem, limit)
           if (result == cutoff) then cutoff-occurred := true;
           else if (result != fail) then return result;
        end for
        if (cutoff-occurred) then return cutoff; else return fail;
end function
```

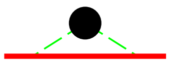
Iterative Deepening Search

```
function Iterative-Deepening-Search (problem) return soln
    for limit from 0 to MAX-INT do
       result := Depth-Limited-Search(problem, limit)
       if (result != cutoff) then return result
    end for
end function
```

Iterative Deepening Search

Limit = 0



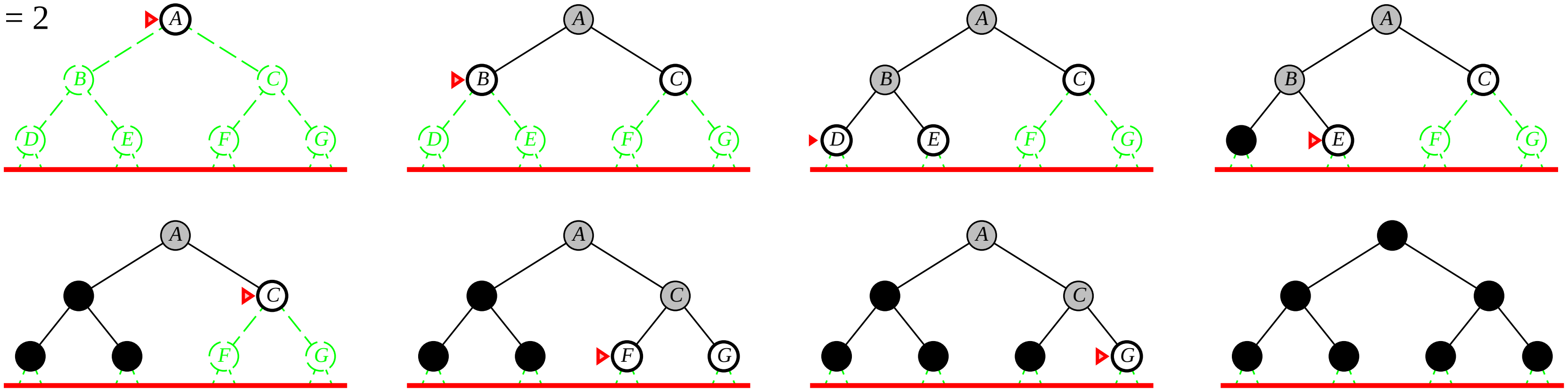


Iterative Deepening Search

Limit = 1

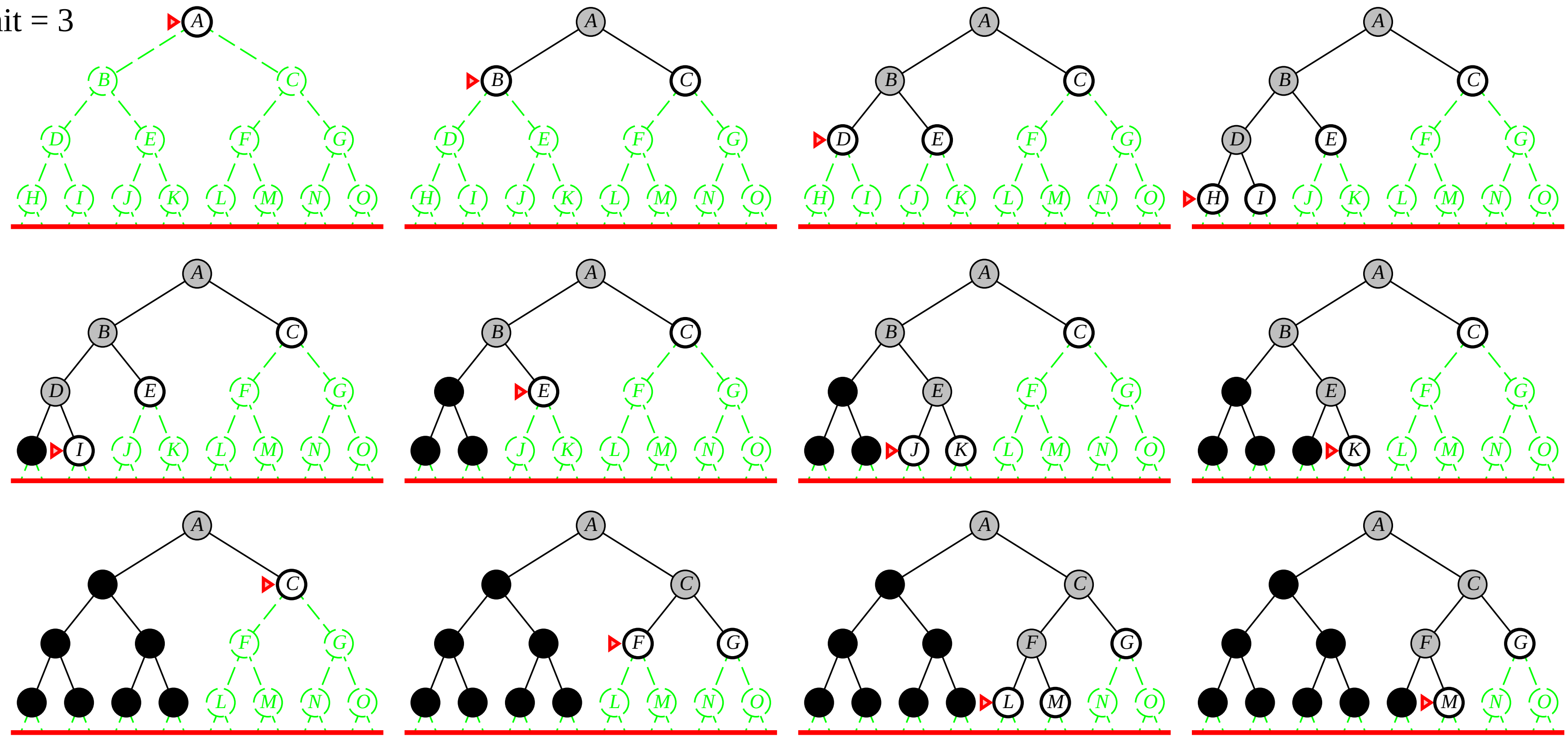
Iterative Deepening Search

Limit = 2



Iterative Deepening Search

Limit = 3



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Properties of Iterative Deepening Search

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Optimal? Only if step costs are all identical

Properties of Iterative Deepening Search

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Optimal? Only if step costs are all identical

Numerical comparison between Iterative Deepening and Breadth First, with $b = 10$, $d = 5$, and solution at "far right" of search tree:

$$
\begin{aligned}
N(\text{ID}) &= 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450 \\
N(\text{BF}) &= 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100
\end{aligned}
$$

Iterative deepening search is actually faster than breadth-first search!

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Iterative deepening search is actually faster than breadth-first search!

It does better because other nodes at depth d are not expanded

Summary of Algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes^a	Yes$^{a}, b$	No	Yes, if $l \geq d$	Yes^a
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes^c	Yes	No	No	Yes^c

b, branching factor d, depth of shallowest solution l, depth limit

m, depth of search tree C^*, cost of optimal solution

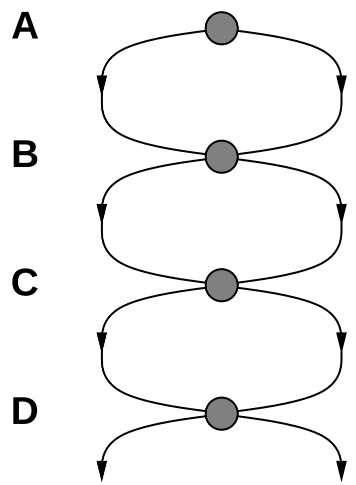
^{a} if b is finite

^{b} is step costs $> \epsilon$ for some $\epsilon > 0$

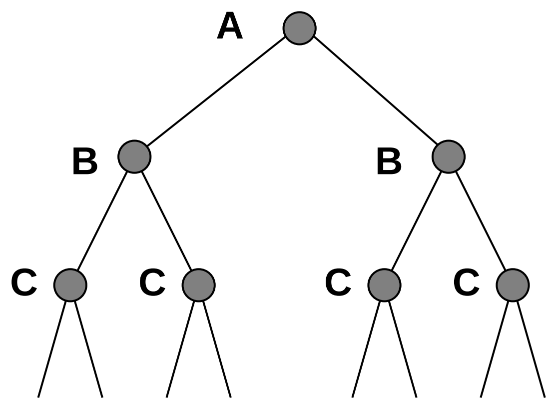
^{c} is all step costs are the same

Repeated States

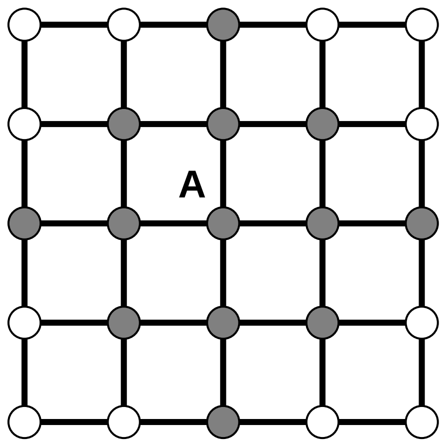
Failure to detect repeated states can turn a linear problem into an exponential one!



(a)



(b)



(c)