

Conversion to CNF

To turn a propositional formula in Conjunctive Normal Form apply to completion each of the following rewrite rules, in the given order.

1. Eliminate implications

$$\alpha \rightarrow \beta \implies \neg\alpha \vee \beta$$

2. Move \neg inwards

$$\begin{aligned}\neg\neg\alpha &\implies \alpha \\ \neg(\alpha \vee \beta) &\implies \neg\alpha \wedge \neg\beta \\ \neg(\alpha \wedge \beta) &\implies \neg\alpha \vee \neg\beta\end{aligned}$$

3. Distribute \vee over \wedge

$$\begin{aligned}\alpha \vee (\beta \wedge \gamma) &\implies (\alpha \vee \beta) \wedge (\alpha \vee \gamma) \\ (\beta \wedge \gamma) \vee \alpha &\implies (\beta \vee \alpha) \wedge (\gamma \vee \alpha)\end{aligned}$$

4. Flatten conjunctions and disjunctions

$$\begin{aligned}(\alpha \wedge (\beta \wedge \gamma)) &\implies (\alpha \wedge \beta \wedge \gamma) \\ (\alpha \vee (\beta \vee \gamma)) &\implies (\alpha \vee \beta \vee \gamma) \\ ((\alpha \wedge \beta) \wedge \gamma) &\implies (\alpha \wedge \beta \wedge \gamma) \\ ((\alpha \vee \beta) \vee \gamma) &\implies (\alpha \vee \beta \vee \gamma)\end{aligned}$$