Two Rolls

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This was done in Mathematica 6.0

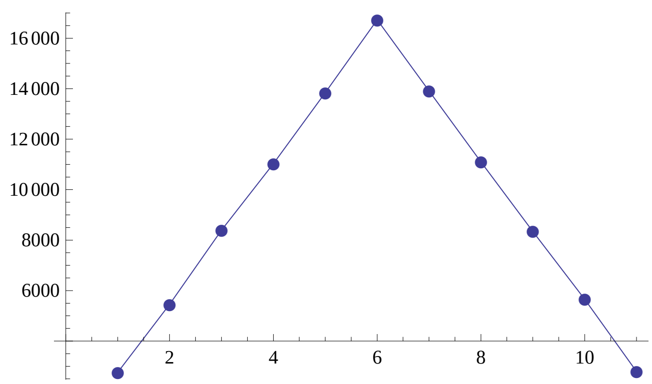
The number of times different sums in the range 2 through 2*n occur, is quite different.
For example, in experiments with a 6-sided dice the sums 2 and 12 occur rarely relative to the sum 7. In general, the sums in the middle of the range 2 through 2*n occur more frequently than the sums at the ends of the range. For an experiment with 6-side dice, I got the following output.

Sum	Number of occurances
2	2758
3	5453
4	8390
5	11028
6	13839
7	16727
8	13911
9	11097
10	8348
11	5655
12	2794

Plotting these numbers gives the following:

In[3]:=
```
ListPlot[{2758, 5453, 8390, 11 028, 13 839, 16 727, 13 911, 11 097, 8348, 5655, 2794},
 PlotJoined → True, PlotMarkers → Automatic]
```

Out[3]=



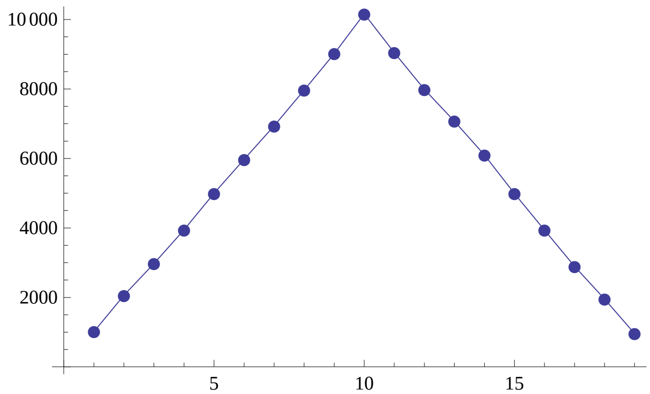
For an experiment with a 10-sided dice, I got the following output . The corresponding plot is shown below.

Sum	Number of occurances
2	1016
3	2060
4	2970
5	3948
6	4996
7	5975
8	6938
9	7967
10	9015

11	10162
12	9050
13	7989
14	7074
15	6102
16	4990
17	3939
18	2896
19	1954
20	959

In[4]:=
```
ListPlot[{1016, 2060, 2970, 3948, 4996, 5975, 6938, 7967, 9015, 10162, 9050, 7989, 7074,
  6102, 4990, 3939, 2896, 1954, 959}, PlotJoined → True, PlotMarkers → Automatic]
```

Out[4]=



The sums do not all occur the same number of times because the number of occurances of a sum x is proportional to the number of ways in which two numbers, both in the range 1 through n, add up to x. The following table illustrates this point.

Sum	Ways in which two numbers in the range 1 through 6 add up to that sum	Number
2	1+1	1
3	1+2, 2+1	2
4	1+3, 2+2, 3+1	3
5	1+4, 2+3, 3+2, 4+1	4
6	1+5, 2+4, 3+3, 4+2, 5+1	5
7	1+6, 2+5, 3+4, 4+3, 5+2, 6+1	6
8	2+6, 3+5, 4+4, 5+3, 6+2	5
9	3+6, 4+5, 5+4, 6+3	4
10	4+6, 5+5, 6+4	3
11	5+6, 6+5	2
12	6+6	1

Since the number of distinct pairs of rolls one can get using two 6-sided dice is 36, the probability of obtaining the sum x = 2, 3, ..., 7 is (x-1)/36 and for x = 8, 9, ..., 12, it is
(13-x)/36. More generally, for n-sided dice, for values of x = 2, 3, ..., n+1, the probability is (x-1)/n*n and for values of x = n+2, n+3, ..., 2n, it is (2n+1 - x)/n*n. Using these formulae, below I calculate the expected number of occurances of each sum, first for n = 6 and then for n = 10, for 100,000 rolls.

In[6]:=
```
Prob[x_, n_] := If[x ≤ n + 1, 100000 * (x - 1) / n^2, 100000 * (2 n + 1 - x) / n^2]
```

In[9]:=
```
Table[Prob[x, 6], {x , 2, 12}] // N
```

Out[9]=
```
{2777.78, 5555.56, 8333.33, 11111.1, 13888.9,
 16666.7, 13888.9, 11111.1, 8333.33, 5555.56, 2777.78}
```

In[10]:=
```
Table[Prob[x, 10], {x , 2, 20}] // N
```

Out[10]=
```
{1000., 2000., 3000., 4000., 5000., 6000., 7000., 8000., 9000.,
 10000., 9000., 8000., 7000., 6000., 5000., 4000., 3000., 2000., 1000.}
```

So for n = 6, the theoretically predicted number of occurances and the experimentally obtained number of occurances is given below.

Sum	Experimental Result	Theoretical Prediction
2	2758	2777.78
3	5453	5555.56
4	8390	8333.33
5	11028	11111.1
6	13839	13888.9
7	16727	16666.7
8	13911	13888.9
9	11097	11111.1
10	8348	8333.33
11	5655	5555.56
12	2794	2777.78

As you can see from the above table the experimental results match up the theoretical prediction quite closely (less than 1% difference). The same is true for n = 10.