Lecture Notes: Social Networks: Models, Algorithms, and Applications
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1 Introduction

Innovation, perhaps unfortunately, can take a long time to be adopted within a community. For example, the prehistoric disease scurvy is recognized in modern times to be caused by a vitamin C deficiency. When naval technology advanced to the point of making long voyages possible, crews began to show symptoms of the disease from the altered diet that comes with long trips. Without a known treatment many sailors died from scurvy: 116 of 170 men of Vasco de Gama's 1497 voyage; 208 of 230 men of Magellan's 1520 voyage; 1300 of 2000 men of Anson's 1720 voyage. In fact, from 1500 to 1800, more sailors died from scurvy than all other disease, disasters, and battles combined.[1]

The first evidence of citrus being used to treat the disease was in 1536 when Jacques Cartier arrived in Newfoundland. The native Indians offered a tea made from spruce tree needles now known to be very high in vitamin C [1]. The remedy cured most of Cartier's crew.

It was not until 1795 that the use of lemon juice was mandated in the royal navy.

Scientist have been interested in this idea of the diffusion of innovation and ideas for a long time. Epidemiologists consider the diffusion of innovation to be directly analogous to the diffusion of disease. In which case social connectedness of the network matters. Sociologists are interested in information cascades, and the diffusion of rumours. In particular, sociologists tend to view humans as rational, but are interested in when we succumb to “herd behaviour”.

2 Diffusion of Hybrid Adoption in Iowa

Ryan and Gross conducted a survey based study in 1943 which examined the time at which farmers adopted a new hybrid seed [2]. The hybrid seeds had the advantage over normal seeds in that the hybrids were draught resistant and stood up nicely, making mechanical harvesting easier. These things together led to a higher yield of product. However, the seeds had the deliberate disadvantage that they were unable to reproduce: farmers would need to buy the new hybrid seeds again each year.

Some of the major findings were that the early adopters of the new hybrid seeds tended to be larger farms with more highly educated farmers: Larger farms are more able to take on increased risk, and more education may contribute to a better understanding of the potential pay off. The average period from first knowledge to adoption was about 9 years they said. Additionally, where you hear about the seeds matters: Someone you know may sway you more than a farming magazine or salesperson.

2.1 Contributions

Of significance was the characterization of “types” of decision makers. Ryan and Gross were able to distinguish five different categories of adopters, each described in their own way:

1. Innovators
2. Early adopters
3. Early Majority
4. Later Majority
5. Laggards

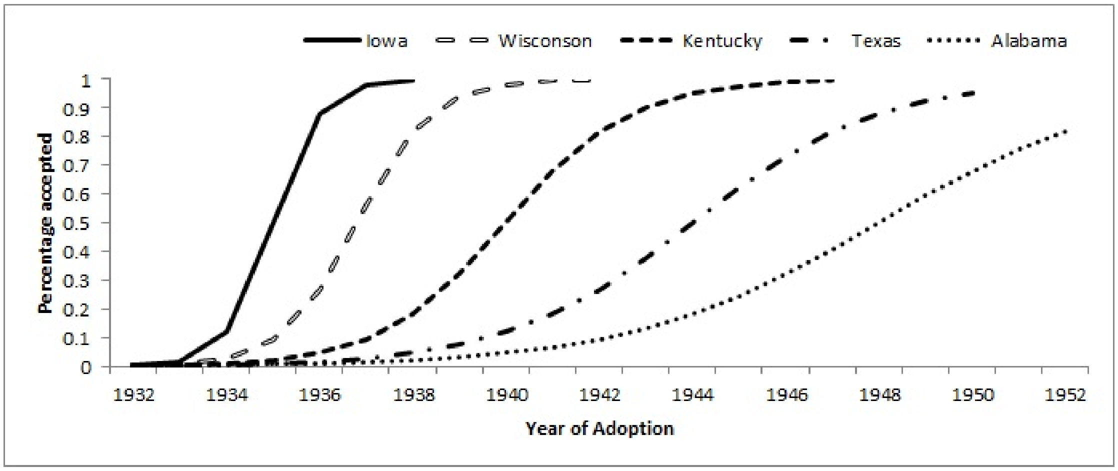
They noted that the difference between each group was attributable to where they heard about the seeds from. Those in the “innovators” group heard about the hybrid seeds mostly from farming magazines and salespersons. Early adopters were convinced from the same, but also from conversations with innovators. The other categories tended to adopt the technology less and less based on outside sources, and more and more on conversations with known adopters. The laggards did not adopt the new technology until everyone was using it.

Ryan and Gross characterized five steps to adoption. These steps outline the progression from just hearing about the innovation (awareness), to a curiosity about it (interest), to investigating it through related articles or conversation with others (evaluation), to actually trying the seed for him or herself (trial), to the conclusion of personally using it and suggesting it to their friends.

1. Awareness
2. Interest
3. Evaluation
4. Trial
5. Adoption

The awareness and evaluation stages give way to two channels of information which an individual may reach conclusions. He or she may speak with a neighbour (local knowledge), or refer to articles published on the subject (global / public knowledge).

In his dissertation, Zvi Griliches expanded the notion of hybrid spread diffusion by investigating the adoption patterns in other agricultural regions of the United States [3]. His research showed that adoption of the new hybrid seed technology was not a single event, but a series of developments. Figure 1 shows an approximation of Griliches findings of how different states adopted the new technology.



3 Models of Adoption

3.1 Random mixing model

Similar to random mixing models, Bass developed a discrete model for adoption [4]. Let $F(t)$ denote the fraction of agents with new behaviour at time t and define parameters p and q as the rate of (spontaneous) innovation and the rate of imitation respectively. Then we can define the recursive relation

$$F(t) = F(t-1) + p(1 - F(t-1)) + q(1 - F(t-1))(F(t-1))$$

with $F(0) = 0$. The multiplication of $1 - F(t-1)$ and $F(t-1)$ in the third term is analogous to the amount of mixing that occurs; a large proportion of both means a lot of mixing and therefore many imitations. Conversely, if one or the other is relatively small, not much mixing occurs and not many imitate. This places a lot of emphasis on p to start the process moving. If $p = 0$, then no adoption occurs in the first place, and consequently no one is ever able to imitate. Additionally, p may be defined as $p(t)$, such that the number of early adopters lessens with time. This accounts for the idea of there being a finite number of innovators, and hence they eventually are all "used up".

Note that this relation is *progressive*: once a behaviour is adopted, it sticks. Additionally, it assumes that spontaneous adoption exists.

Written in continuous form, the rate of adoption has the form

$$\frac{dF(t)}{dt} = (p + qF(t))(1 - F(t))$$

which implies that

$$F(t) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-1(p+q)t}}$$

.

Coleman, Katz, Menzel explored early adopting patterns of doctors [5]. They found that more heavily connected doctors, what they deemed more *cosmopolitan*, were more likely to be early

u v	A	B
A	a,a	0,0
B	0,0	b,b

adopters of the drug tetracycline. Ryan and Gross revisited their initial findings and concluded that early adopting farmers were similarly more cosmopolitan. Farmers that lived near big cities, Des Moines in Iowa for example, were more likely the innovators.

3.2 Agent-based model

The random mixing model shows the expected proportion of adoption. This however can be a greatly limiting factor and many similar models have been abandoned in favour of agent-based model that hold inherit network structure. Consider a graph $G = (V, E)$ with vertices as those that may adopt the behaviour and edges indicating connections between them (such as neighbours or close friends). Initially, the graph starts with a predefined A number of "seeds" or nodes that are considered early adopters. Individuals can than choose whether to accept a new behaviour according to the pay off matrix.

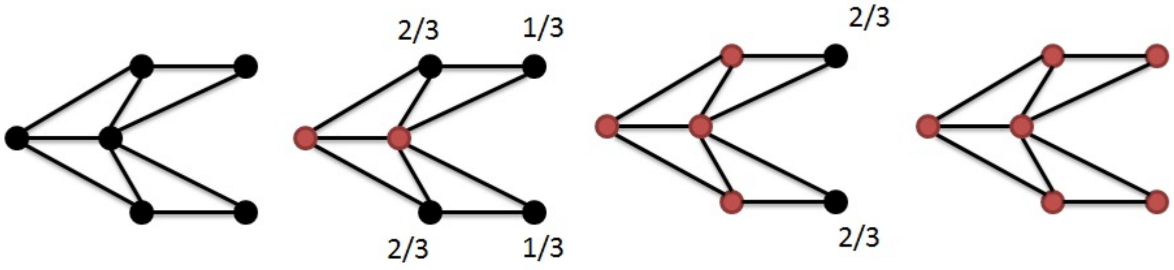
This matrix implies that for an edge $\{u, v\} \in E$, and both have behaviour A, then node u gets pay off a and node v gets pay off a. Conversely, if both have behaviour B, then node u gets pay off b and node v gets pay off b. If their behaviour is different than neither node receives pay off. If each node behaves rationally, he or she will choose the behaviour that maximizes their pay off.

This leads to the notion of a threshold for a particular node. That is, there is a certain tipping point at which it is in the node's interest to change behaviour. Assuming a node starts with behaviour B and has d neighbours, then the node will switch to behaviour A when

$$p \cdot d \cdot a \geq (1 - p) \cdot d \cdot b$$

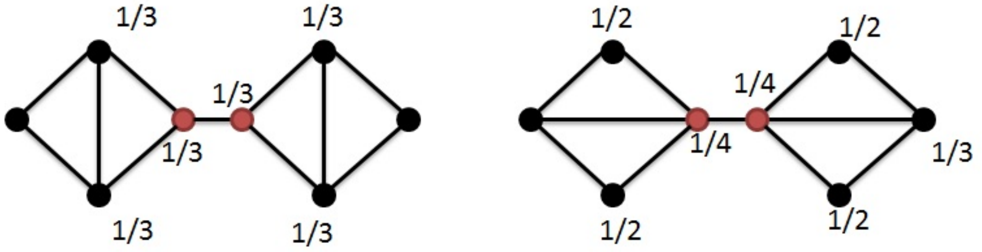
for a threshold p. Solving for p we find that the node will switch behaviours if $p \geq \frac{b}{a+b}$ neighbours are behaviour A. Note that at the boundaries of $p \approx 1$ the node has no incentive to switch, and $p \approx 0$ he or she will switch relatively easily (even only 1 neighbour may suffice).

Consider the example graph in Figure 2(a), with pay off of $a = 3$ and $b = 2$. The threshold of changing behaviour then is $p \geq \frac{2}{5}$. Some nodes are initially given the new behaviour (considered early adopters or innovators) and are colored red as seen in Figure 2(b). When this happens, some of the nodes have $\frac{2}{3}$ of their neighbours with the new behaviour, others only $\frac{1}{3}$. The nodes with $\frac{2}{3}$ of their neighbours change behaviour on the next time step as seen in Figure 2(c) since $\frac{2}{3} \geq \frac{2}{5}$, the calculated threshold . Now, the nodes that originally only had $\frac{1}{3}$ of neighbours with changed behaviour, have $\frac{2}{3}$ of their neighbours with different behaviour. So then on the next time step, all the nodes have changed behaviour.



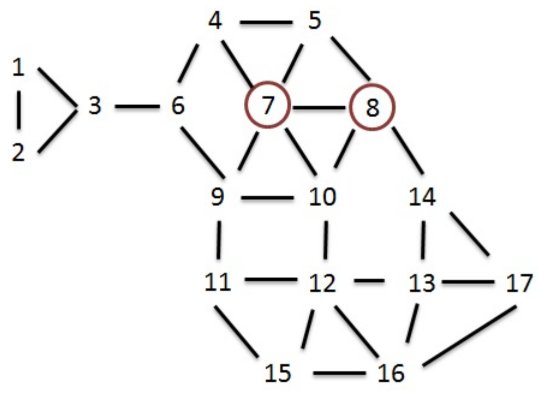
(a) (b) (c) (d)

It is also not abundantly clear that a certain behaviour will necessarily "catch on". In fact, the network structure might inhibit the adoption of innovation in the entire network. With the same pay off of $a = 3$ and $b = 2$, Figure 3 shows two possible networks. In (a), none of the nodes have greater than the $\frac{2}{5}$ neighbour threshold to change behaviour and so the behaviour is only adopted by a subset of nodes. In (b), changing two edges changes the number of neighbours that have adopted the new behaviour for a few nodes, and so will change their own behaviour on the next time step. It is clear that the new technology will spread through the network.

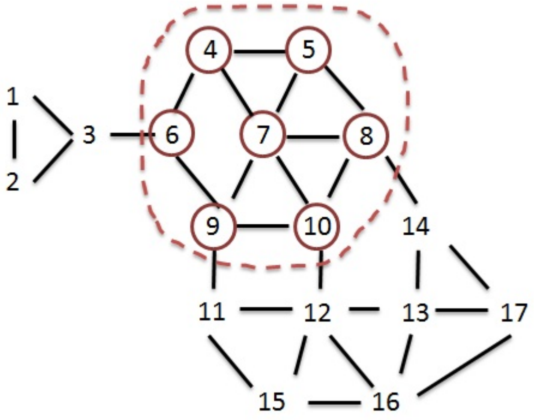


(a) (b)

In general, a "tight" community can disrupt the propagation of change. For example, consider the network in Figure 4(a) where nodes 7 and 8 initially adopt a behaviour. While many of their neighbours also adopt the behaviour, the innovation does not diffuse through the entire network since some nodes never exceed the threshold.



(a)



(b)

3.3 Extensions

One question asked by product marketers is, "When propagation stops, who should I target to restart it?". This question, perhaps not surprising, is NP-hard and therefore unlikely to have a solution this is easily attainable. Additionally, one might consider other variations of the above models and postulate what effects might be on propagation:

- *Custom payoffs* Instead of a uniform pay off matrix where each node has the same pay off, each node has a personal pay off from being in the behaviour as a specific neighbour node.
- *Custom threshold*
- *Independent cascade models* A model in which a neighbour only in influences a node at the time of its transition.
- *Public and private models* A game theoretic perspective in which not all information is public may change propagation and diffusion results.

References

[1] Jason Allen Mayberry *Scurvy and Vitamin C*. Food and Drug Law, Winter 2004, URL: http://leda.law.harvard.edu/leda/data/658/Mayberry.html, Accessed: 3/7/2012

[2] Bryce Ryan, Neal Gross *The Diffusion of Hybrid Seed Corn in two Iowa Communities*. 1943. Rural Sociology (8) 15-24

[3] Zvi Griliches *Hybrid Corn: An Exploration of the Economics of Technological Change*. 1957. New York: Basil Blackwell

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[5] James Coleman, Herbert Menzel, Elihu Katz *Social Processes in Physicians' Adoption of a New Drug* 1959. Journal of Chronic Diseases 9(1) 1-19