Chapter 1 Review

- Linear equations.
- Geometrical interpretation.
- Number of solutions of a linear system.
- Solving linear systems by row reductions.
- Row-echelon form and reduced row-echelon form of a matrix.
- Matrix operations (addition, scalar multiplication, multiplication including block multiplication) and their properties; matrix transpose.
- Matrix inverse: elementary matrices, finding matrix inverse, using inverses in solving linear system.
- Invertible Matrix Theorem.
- Scalar, diagonal, triangular and symmetric matrices and their properties.

1. Mark each statement either True or False. Justify your answer.

(a) If a matrix is reduced to reduced row-echelon form by two different sequences of elementary row operations, the resulting matrices will be different.

(b) Every matrix is row equivalent to a unique matrix in row-echelon form.

(c) If three lines in the $xy$-plane are sides of a triangle, then the system formed from their equations has three solutions, one corresponding to each vertex.

(d) A linear system of three equations in five unknowns must be consistent.

(e) A linear system of five equations in three unknowns cannot be consistent.

(f) If a system of linear equations has two different solutions, then it has infinitely many solutions.

(g) If a system $Ax = b$ has more than one solutions, then so does the system $Ax = 0$.

(h) $Ax = 0$ implies $x = 0$.

(i) If matrices $A$ and $B$ are row equivalent then they have the same reduced echelon form.
(j) If $A$ is an $m \times n$ matrix and if the equation $Ax = b$ has a solution for every $b$ in $\mathbb{R}^m$, then $A$ has $m$ pivot columns.

(k) If an $m \times n$ matrix $A$ has a pivot position in every row, then the equation $Ax = b$ has a solution for every $b$ in $\mathbb{R}^m$.

(l) If an $n \times n$ matrix $A$ has $n$ pivot positions, then the reduced echelon form of $A$ is the $n \times n$ identity matrix.

(m) Every square matrix can be expressed as a product of elementary matrices.

(n) The product of two elementary matrices is an elementary matrix.

(o) If $A$ is invertible and $AB = O$, then it must be true that $B = O$.

2. Let $A$ and $B$ be $n \times n$ matrices. Indicate whether the following statements are always true or sometimes false:

(a) $(AB)^2 = A^2B^2$.

(b) $(AB^{-1})(BA^{-1}) = I_n$.

(c) $(A + B)^2 = A^2 + 2AB + B^2$.

3. Find a formula for the product of two diagonal matrices.

4. Use block product to find the product $AB$ of the following matrices

\[
A := \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 2 & 0 \\
0 & 0 & 0 & 2
\end{bmatrix}, \quad B := \begin{bmatrix}
1 & 2 \\
3 & 4 \\
1 & 0 \\
0 & 1
\end{bmatrix}.
\]