2.1

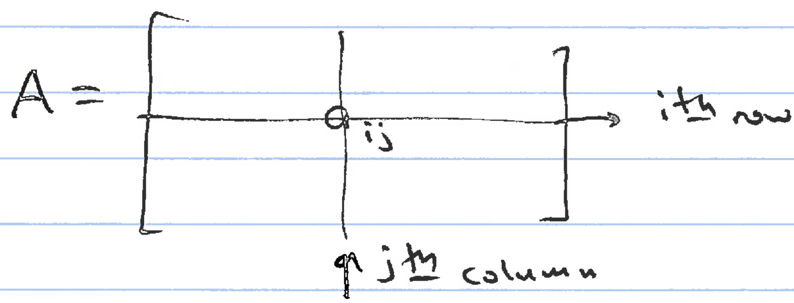
Defn An $m \times n$ matrix A is an array of mn entries written in m rows & n columns.

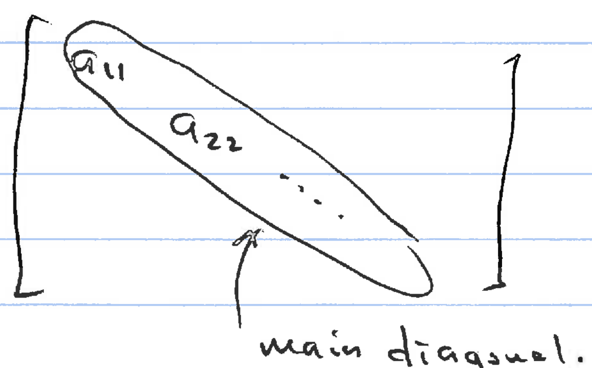
$$\begin{array}{cc} & \begin{array}{ccc}\downarrow & \downarrow & \downarrow\end{array}\ \text{3 columns.} \\ \text{2 rows} \begin{array}{c}\to \\ \to\end{array} & \begin{bmatrix} 2 & 1 & 4 \\ 3 & -1 & 0 \end{bmatrix} \end{array}$$

2×3 matrix.

$$A = \begin{bmatrix} \vec{a}_1 & \cdots & \vec{a}_n \end{bmatrix}$$

each is a vector in $\mathbb{R}^m$





$$\begin{bmatrix} 2 & 3 & 7 \\ 1 & 9 & -1 \\ 0 & 6 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Diagonal matrix:
- Square matrix: $n \times n$

and
- All entries off-the main diagonal are 0.

(0 can/may appear on the main diagonal).

<u>Identity matrix</u>
- Diagonal matrix
- All main diagonal entries are 1

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_4.$$

- Scalar Multiples of matrices

$$2 \cdot \begin{bmatrix} 2 & -1 & 4 \\ 7 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 8 \\ 14 & 4 & 0 \end{bmatrix}$$

<u>Defn</u> Let A be an $m \times n$ matrix, $r \in \mathbb{R}$

<u>Define</u> $(rA)_{ij} = r A_{ij}$

In English: the i^{th} row j^{th} column entry of rA is obtained by multiplying r with the i^{th} row j^{th} column entry of A.

Defn Let A and B be both $m \times n$ matrices $\underbrace{(\text{same size})}_{m \times n}$, then

$$(A+B)_{ij} = A_{ij} + B_{ij}$$

In English: How you write it out.

Ex
$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 4 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 & 3 \\ 6 & 4 & 11 \end{bmatrix}.$$

Ex
$$4\begin{bmatrix} -1 & 4 \\ 0 & 6 \end{bmatrix} - 3\begin{bmatrix} 1 & -2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -7 & 22 \\ -6 & 12 \end{bmatrix}$$

Matrix Multiplication

<u>Recall</u>

$$\underset{m\times n}{A}\cdot \underset{\mathbb{R}^n}{\vec{c}} = \begin{bmatrix} \vec{a}_1 & \cdots & \vec{a}_n \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

$$= c_1\cdot \vec{a}_1 + c_2\vec{a}_2 + \cdots + c_n\vec{a}_n$$

$$\underset{3\times 2}{\begin{bmatrix} 2 & 4 \\ 3 & 7 \\ 1 & -1 \end{bmatrix}}\underset{\mathbb{R}^2}{\begin{bmatrix} -1 \\ 2 \end{bmatrix}} = -1\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 7 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ -1 \end{bmatrix} + \begin{bmatrix} 8 \\ 14 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 11 \\ -3 \end{bmatrix}.$$

<u>Defn</u> Let A be an $m\times n$ matrix & B be an $p\times q$ matrix.

- If $n \neq p$, then AB is not defined
- If $n=p$, then AB is defined to be a

$$(m\times n)\cdot(p\times q) \longrightarrow m\times q \text{ matrix}:$$

$$AB = A\cdot\begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \cdots & \vec{b}_q \end{bmatrix} = \begin{bmatrix} A b_1 & A b_2 & & A b_q \end{bmatrix}$$

each calculated as above

Calculate by using defn, in the long way

Ex $$\begin{bmatrix} -2 & 1 & 4 \\ 5 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ -1 & 1 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$$

$$\underbrace{2 \times 3 \quad 3}_{=} \times 2 \longrightarrow 2 \times 2$$

$$= \underbrace{\begin{bmatrix} -2 & 1 & 4 \\ 5 & 0 & 6 \end{bmatrix}}_{A} \begin{bmatrix} \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} & \begin{pmatrix} 6 \\ 1 \\ 4 \end{pmatrix} \end{bmatrix}$$

$$\vec{b}_1 \qquad \vec{b}_2$$

Long way

$$= \Bigg[\underbrace{\begin{bmatrix} -2 & 1 & 4 \\ 5 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}}_{\text{first column}} \quad \underbrace{\begin{bmatrix} -2 & 1 & 4 \\ 5 & 0 & 6 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \\ 4 \end{bmatrix}}_{\text{second column}} \Bigg]$$

$$\begin{bmatrix} -2 & 1 & 4 \\ 5 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} = 1 \begin{bmatrix} -2 \\ 5 \end{bmatrix} + (-1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 4 \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 5 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 16 \\ 24 \end{bmatrix}$$

$$= \begin{bmatrix} 13 \\ 29 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 4 \\ 5 & 0 & 6 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \\ 4 \end{bmatrix} = 6 \begin{bmatrix} -2 \\ 5 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 4 \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 5 \\ 54 \end{bmatrix}$$

Same Ex Short method

$$\begin{bmatrix} -2 & 1 & 4 \\ 5 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ -1 & 1 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 13 & 5 \\ 29 & 54 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} -2 & 1 & 4 \end{bmatrix}}_{\text{First row of } A} \underbrace{\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}}_{\text{First column of } B} = (-2 \cdot 1) + (1 \cdot -1) + (4 \cdot 4)$$

$$= -2 + -1 + 16 = \underbrace{13}_{\text{1st row 1st column entry of } AB.}$$

Row-Column Rule

- Let A be an $m \times n$ matrix,
- Let B be a $p \times q$ matrix, and
- $p = n$.

Then AB is a $m \times q$ matrix whose

ith row jth column entry : $(A \cdot B)_{ij}$

is the product of the ith row of A
& jth column of B

which is obtained by taking the sum of the products of the corresponding entries as follows:

2/17/14

p7

$$(A\cdot B)_{ij} = \underbrace{[a_{i1}\ a_{i2}\ \cdots\ a_{in}]}_{i^{th}\text{ row of }A} \underbrace{\begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{nj} \end{bmatrix}}_{j^{th}\text{ column of }B}$$

$$= a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ik}b_{kj} + \cdots + a_{in}b_{nj}$$

$$= \sum_{k=1}^{n} a_{ik}b_{kj} = (A\cdot B)_{ij}$$

Examples: ①

$$\underset{2\times 3}{\begin{bmatrix} -2 & 1 & 4 \\ 5 & 0 & 6 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 1 & 6 \\ -1 & 1 \\ 4 & 4 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} 13 & 5 \\ 29 & 54 \end{bmatrix}}$$

$$\underset{3\times 2}{\begin{bmatrix} 1 & 6 \\ -1 & 1 \\ 4 & 4 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} -2 & 1 & 4 \\ 5 & 0 & 6 \end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix} 28 & 1 & 40 \\ 7 & -1 & 2 \\ 12 & 4 & 40 \end{bmatrix}}$$

So: $AB \neq BA$

Ex 2

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 11 & 7 \end{bmatrix}$$

Again $AB \neq BA$

Ex 3 $\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

False in general.

$AB = 0 \not\Rightarrow A = 0$ or $B = 0$

Caution: Typo Corrected

$AB = AC \not\Rightarrow A = 0$ or $B = C$

Ex 4 $\begin{bmatrix} 1 & 1 & 2 \\ -1 & 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & -1 & 4 \\ 1 & 1 & 2 \end{bmatrix}$ not defined

$\underbrace{2\times 3 \quad 2\times 3}_{\neq}$

Ex $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ $A^2 = A\cdot A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 7 & 6 \\ 9 & 22 \end{bmatrix}$$

Ex

Called Transpose

$$\begin{bmatrix} 1 & 4 \\ 2 & -7 \\ 3 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -7 & 6 \end{bmatrix}$$

$$(3 \times 2)^T = 2 \times 3$$