1.5 Continue

Ex

$$\left.\begin{aligned} x \quad\quad + z &= 0 \\ 2x - y + z &= 0 \\ 5x - 2y + 3z &= 0 \end{aligned}\right\}$$ Homogenous

$\Rightarrow$ $x = y = z = 0$

a Sol'n

Does there exist any other solution?

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 2 & -1 & 1 & 0 \\ 5 & -2 & 3 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

free

$x + z = 0$ $\qquad$ $x = -z$

$y + z = 0$ $\qquad$ $y = -z$

$z = z$ free

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -z \\ -z \\ z \end{bmatrix} = z \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

Vector parametric sol'n.

gives us non-trivial solutions by taking $z \neq 0$

Thm A homogenous SLE $A \cdot \vec{x} = \vec{0}$ has a non trivial solution $\Longleftrightarrow$

The RREF of A has a column without a leading 1

(Not augmented)

Consequences: $A\vec{x} = \vec{0}$

① Every column of RREF of A has a leading 1

$\Longleftrightarrow$ $x = 0$ is the only solution

② The number of parameters is the # of the columns of RREF(A) without a leading 1.

③ In All cases, the solution set of $A\vec{x} = \vec{0}$ is the span ~~of~~ a finite set of vectors.

If $x=0$ the only soln $\Rightarrow$ $\{\vec{0}\} = \text{span}\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right)$

Non-homogeneous SLES. $A \cdot \vec{x} = \vec{b}$ $(b \neq 0)$

- May or may not be consistent
-

Ex $\left.\begin{aligned} 2x + y + z &= 3 \\ x + y - z &= 2. \end{aligned}\right\} (*)$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 1 & 1 & -1 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 2 & 1 & 1 & 3 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -1 & 3 & -1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -3 & 1 \end{array}\right].$$

free

$x = 1 - 2z$
$y = 1 + 3z$
$z =$ free z

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 - 2z \\ 1 + 3z \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \underbrace{z \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}}.$$

A Particular soln of (*)

This part solves

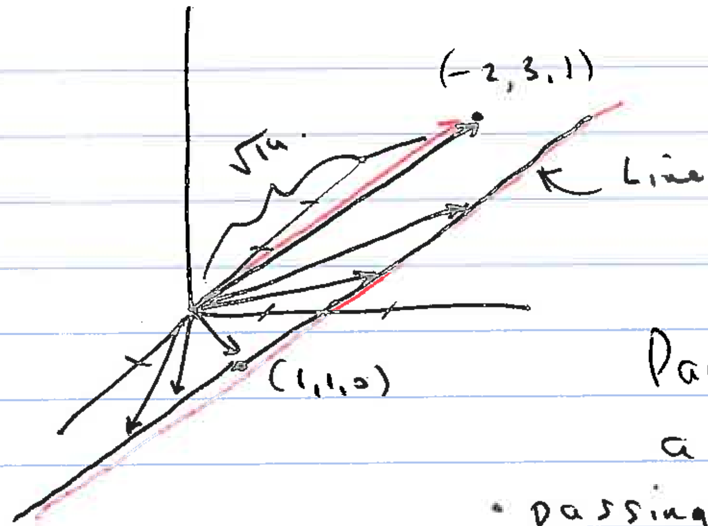
$2x + y + z = 0$

$x + y - z = 0$

2/10/14

p4

$$\begin{bmatrix}1\\1\\0\end{bmatrix} + z\begin{bmatrix}-2\\3\\1\end{bmatrix}$$



Parametric eqn of a line

- passing thru (1,1,0)
- Parallel to $\begin{bmatrix}-2\\3\\1\end{bmatrix}$.

In general:

$\vec{x} = p_0 + t v_0$ is a parametric line

passing thru p_0 and parallel to v_0.

Caution the tips of the vectors $p_0 + t v_0$ are on a line, not the vectors themselves!

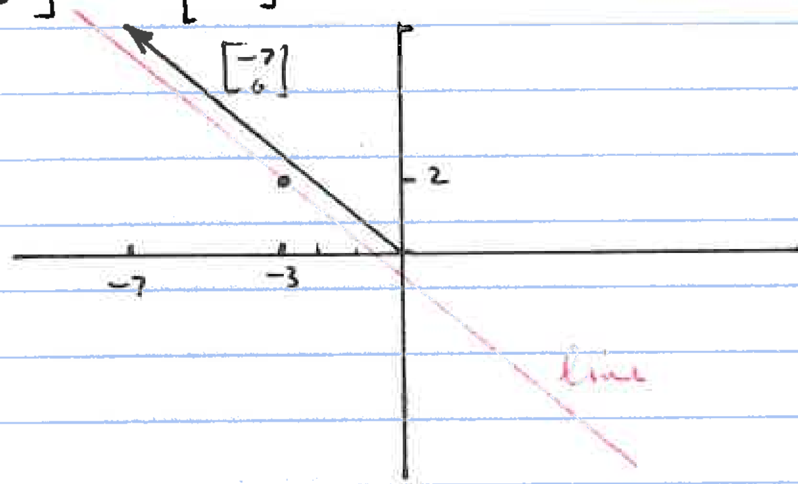
2/10/14.

p5

passing thru $\qquad$ parallel to.

p47 #20 $\quad p = \begin{bmatrix} -3 \\ 2 \end{bmatrix}, \; a \begin{bmatrix} -7 \\ 6 \end{bmatrix}$

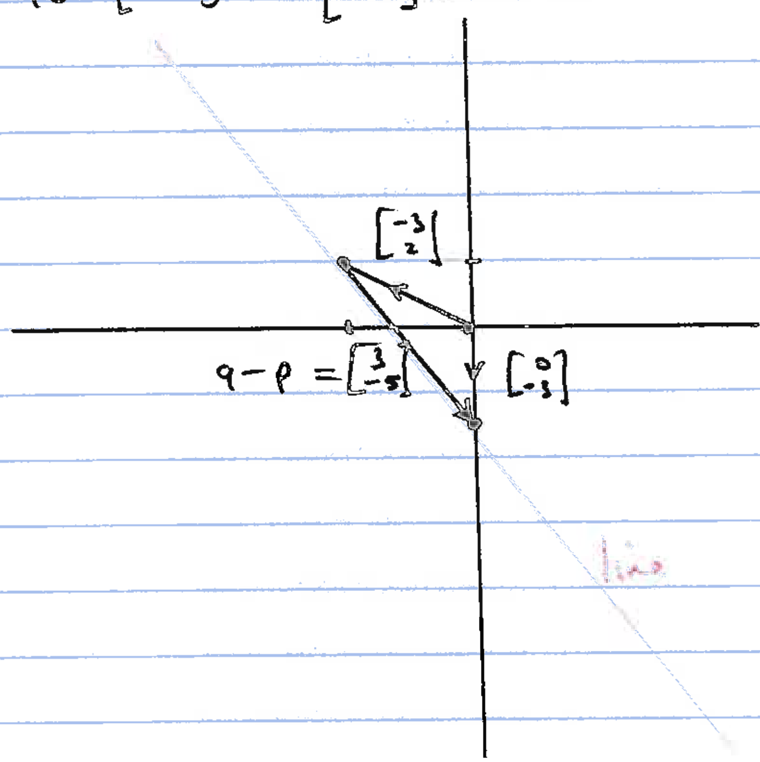
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix} + t \begin{bmatrix} -7 \\ 6 \end{bmatrix}$$



p47 #22 $\quad$ Passing thru $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -3 \end{bmatrix}$.

$$\vec{x} = p + t(q-p)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix} + t \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$



Thm 6 p 46

Let $A\vec{x} = \vec{b}$ be given SLE

Let

① p_0 be a (particular) solution of $A\vec{x} = \vec{b}$

that is: $A \cdot \vec{p_0} = \vec{b}$

② Let V_h describe a parametric (& complete) solution of $A\vec{x} = \vec{0}$.

Then $p_0 + V_h$ describes the complete solution of $A \cdot x = b$.

Later in **M34** for ODE's

PTO

Ex 2 ⊛ $x - y - 2z = -2$. (A plane?)

⊙ A particular soln : $(0,0,1)$

⊙ Solve $x - y - 2z = 0$

$$\begin{array}{cccc} \left[\,1\right. & -1 & -2 & \left|\,0\,\right] \\ & \uparrow & \uparrow & \\ & \text{free} & \text{free} & \end{array}$$

$x - y - 2z = 0$ ①

$x = y + 2z$

$y = \text{free } y$

$z = \text{free } z$

$$V_h = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y + 2z \\ y \\ z \end{bmatrix} = \underbrace{y \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}}_{V_h}.$$

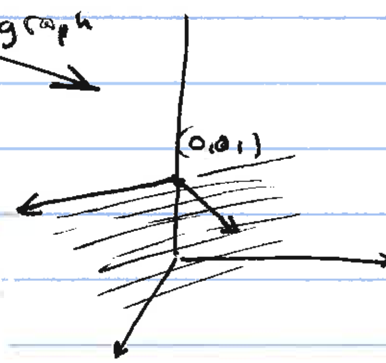
This one solves homogenous ①

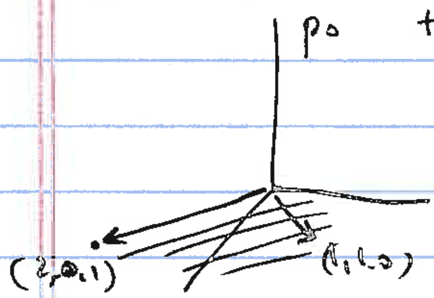
Now : Soln of $x - y - 2z = -2$?

solves ⊛

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{p_0} + \underbrace{y \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}}_{V_h}$$

graph





Shift by $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. ↑

Soln of $x - y - 2z = 0$ ①

Soln of $x - y - 2z = -2$.