1/29/14 (p1)

RREFS

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 7 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & 0 & 3 \end{array}\right]$$ Inconsistent

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow$$ Consistent since there exists no row $[0 \dots 0 \mid c \neq 0]$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 1 & -1 & 7 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ Inconsistent

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THM Every matrix is row equivalent to a unique RREF (reduced row echelon form)

SLE

$$\left[\begin{array}{c|c} \text{coeff matrix} & \text{augmentation} \end{array}\right]$$

$$\underbrace{\underbrace{\text{coeff matrix}}_{\substack{n \text{ unknowns} \\ n \text{ columns}}} \quad \underbrace{\text{augmentation}}_{1 \text{ column}}}_{n+1 \text{ columns.}}$$

row reduce

$$\left[\begin{array}{c|c} \text{Echelon form} & \begin{matrix} \vdots \\ \vdots \end{matrix} \end{array}\right]$$

~~Here~~ Decide consistent or not (i.e. inconsistent)

If $\exists$ row $[0 \ldots 0 \mid c \neq 0]$

stop if inconsistent & state it so.

If consistent

row reduce

$$\left[\begin{array}{c|c} \text{RREF} & \begin{matrix} c_1 \\ \vdots \\ c_m \end{matrix} \end{array}\right]$$

PTO

Case 1

$$\left[\begin{array}{cccc|c} \multicolumn{4}{c|}{\text{coeff. matrix}} & \\ 1 & & 0 & & c_1 \\ & 1 & & & \vdots \\ 0 & & 1 & & \\ 0 & 0 & 0 \cdot & 1 & c_n \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

augmentation

RREF

n columns $\Leftrightarrow$ n unknowns

Every column has a leading 1

Then unique soln

$$\left[\begin{array}{l} x_1 = c_1 \\ x_2 = c_2 \\ \vdots \\ x_n = c_n \end{array}\right.$$

Case 2 There are some columns of RREF of the main coeff matrix (not including the augmentation) without a leading one of any row.

$$\left[\begin{array}{cccc|c} 1 & 0 & * & 0 & \vdots \\ 0 & 1 & * & 0 & \vdots \\ 0 & 0 & 0 & 1 & \vdots \\ 0 & 0 & 0 & 0 & \vdots \\ 0 & \vdots & & 0 & \vdots \\ 0 & & 0 & \cdot & \end{array}\right]$$

No leading 1 in this column

assign free variable

- Do this for each column without a leading 1 of a row.
- Not for augmentation

Then solve all of the unknowns in terms of the free variables

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$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \end{array} \\ \left[\begin{array}{ccccc|c} 1 & 0 & -5 & 0 & 0 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$8R_3 + R_1 \to R_1$

given $\left[\begin{array}{ccccc|c} 1 & 0 & -5 & 0 & -8 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

x_3 free

x_4 free

$$\begin{cases} x_1 - 5x_3 = 3 \\ x_2 + 4x_3 - x_4 = 6 \\ x_5 = 0 \end{cases}$$

Soln:

$x_1 = 3 + 5x_3$

$x_2 = 6 - 4x_3 + x_4$

$x_3 = x_3$ (free to choose)

$x_4 = x_4$ (free to choose)

$x_5 = 0$

1.3

Defn

- $\mathbb{R}^n = \left\{ \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} \middle| \; c_1, c_2 \dots c_n \in \mathbb{R} \right\}$

- $\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} + \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} c_1 + d_1 \\ c_2 + d_2 \\ \vdots \\ c_n + d_n \end{bmatrix}$ vector addition

- $k \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} kc_1 \\ kc_2 \\ \vdots \\ kc_n \end{bmatrix}$ scalar multiple ~~of a~~ vector.

Ex

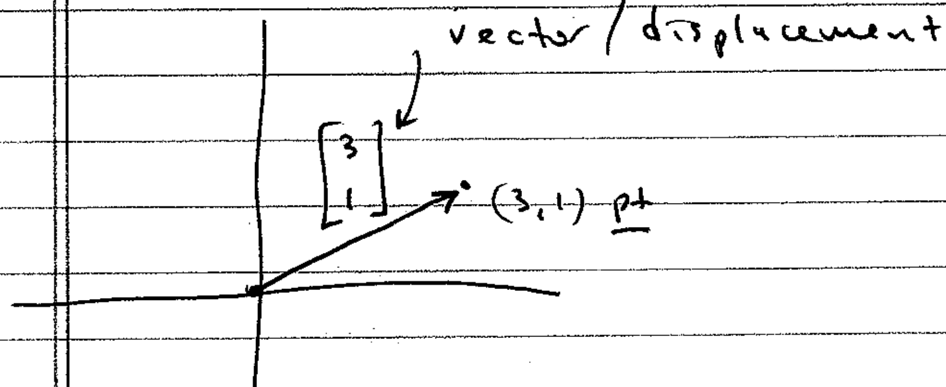
$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 2-1 \\ 3+4 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}$ Can't Add!

$$5 \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \\ 15 \end{bmatrix}$$

$$\underbrace{7\begin{bmatrix}2\\-3\\4\end{bmatrix}+(-1)\begin{bmatrix}4\\-1\\4\end{bmatrix}}_{\text{linear combination.}}=\begin{bmatrix}14\\-21\\28\end{bmatrix}+\begin{bmatrix}-4\\1\\-4\end{bmatrix}=\begin{bmatrix}10\\-20\\24\end{bmatrix}$$

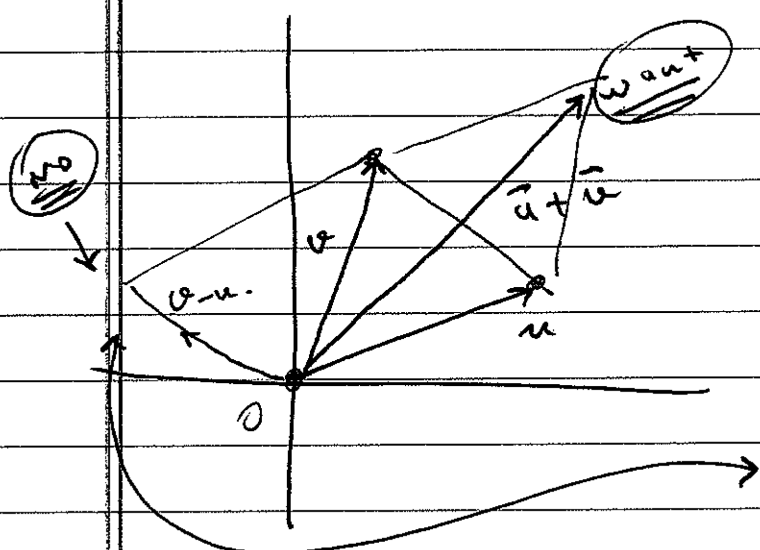
<u>Geometric Discussion</u>



$\begin{bmatrix}a\\b\end{bmatrix}$ describes a displacement from $\vec{0}$ to the point (a, b)

Similarly for $\mathbb{R}^3$.

Parallogram Law:



To avoid this case

If one places a parallogram whose three vertices are the terminal pts of the vectors $\vec{u}, \vec{0}, \vec{v}$, s.t. the terminal points of $\vec{u}$ and $\vec{v}$ are not adjacent on the parallogram,
then: the 4^{th} vertex is the terminal pt of $\vec{u}+\vec{v}$

This rule extends to $\mathbb{R}^3$ if we define a parallogram in $\mathbb{R}^3$ to be a quadrilateral whose opposite sides are parallel. Such parallograms are co-planar.

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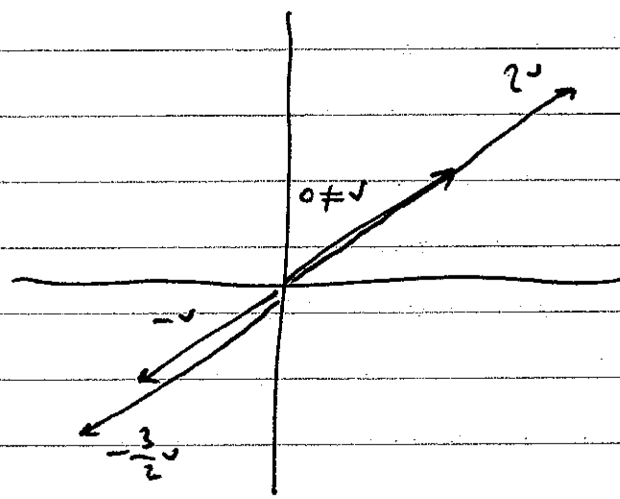
Linear Combinations

Defn Given vectors $\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k$ in $\mathbb{R}^n$
k real #'s $c_1, c_2, \ldots c_k$ in $\mathbb{R}$, one defines

$$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 + \cdots + c_k\vec{v}_k$$ to be the linear combination of $v_1, v_2, \ldots v_k$ with weights $c_1, c_2 \ldots c_k$.

Geometrically



The terminal points of the vectors $k\vec{v}$ where $\vec{v} \neq 0$ is fixed, ~~and~~ k varying thru $\mathbb{R}$, lie on a line, passing thru $\vec{0}$ and the terminal pt of $\vec{v}$.

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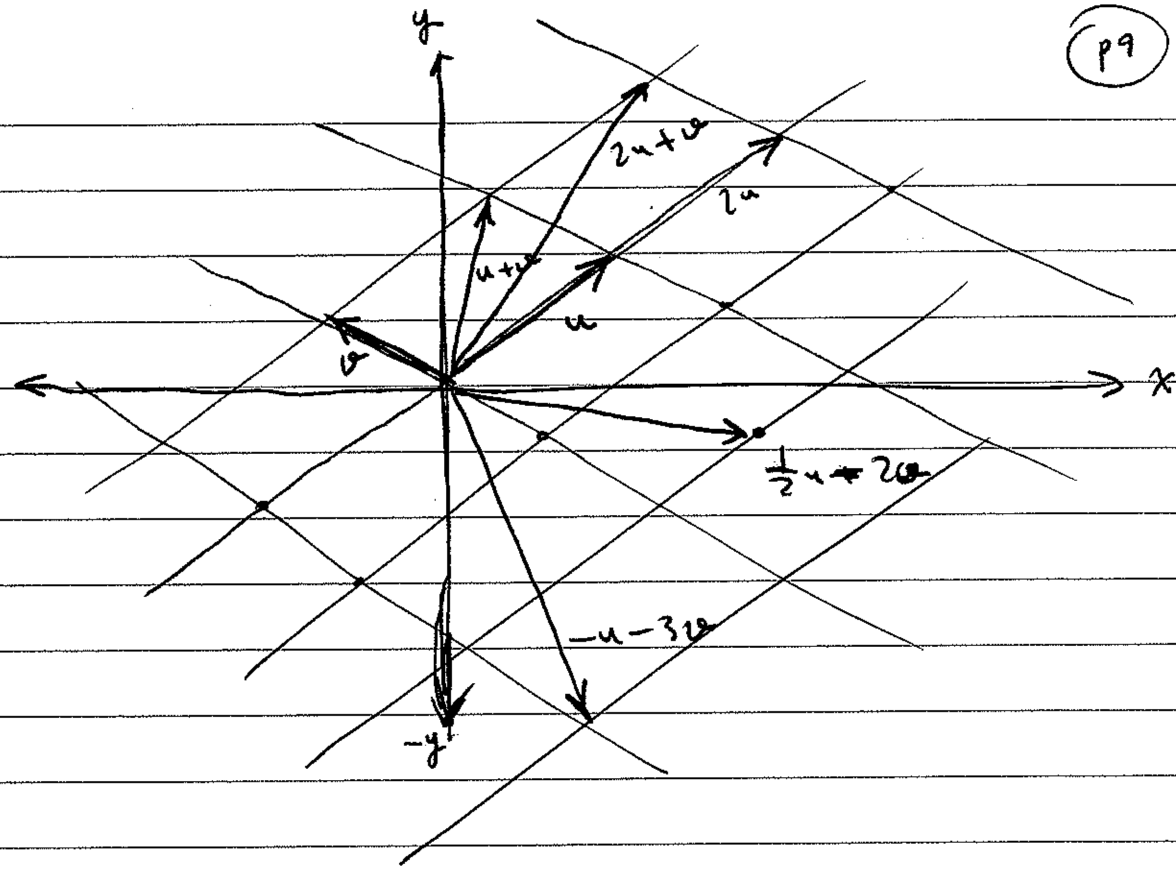

The linear combinations of the form

$c_1\vec{u} + c_2\vec{v}$ (where $u \neq 0$, $v \neq 0$, u is not parallel to v).

with some chosen values of c_1 and c_2.