$$\underline{SLE} \longrightarrow \left[\begin{array}{c|c} \text{Coeff Matrix} & \text{Augmentation} \end{array}\right]$$

Elementary Row operations:

- Add a multiple of a row to another
- Interchange 2 rows
- Multiply a row with a non-zero real #.

Defn 2 SLE's are called row-equivalent if there is a finite sequence of elementary row operations which takes one matrix of augmented coeff matrix to the other.

Thm: If 2 SLE'S are row-equivalent then they have the same solutions.

Ex
$$\left.\begin{aligned} x+y&=5 \\ x-y&=3 \\ 2x+y&=3 \end{aligned}\right\} \text{ solve.}$$

$$\left[\begin{array}{cc:c} 1 & 1 & 5 \\ 1 & -1 & 3 \\ 2 & 1 & 3 \end{array}\right] \xrightarrow[R_2 - R_1 \to R_2]{} \left[\begin{array}{cc:c} 1 & 1 & 5 \\ 0 & -2 & -2 \\ 2 & 1 & 3 \end{array}\right] \xrightarrow[R_3 - 2R_1 \to R_3]{} \left[\begin{array}{cc:c} 1 & 1 & 5 \\ 0 & -2 & -2 \\ 0 & -1 & -7 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 1 \\ 0 & -1 & -7 \end{array}\right] \xrightarrow[R_3 + R_2 \to R_3]{} \left[\begin{array}{cc:c} 1 & 1 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & -6 \end{array}\right]$$

$0 \cdot x + 0 \cdot y = -6$

Inconsistent

No sol^n.

4/27/14 P2

Ex

$x + y + z = 5$

$x - y + z = 3$

$x + z = 4$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & -1 & 1 & 3 \\ 1 & 0 & 1 & 4 \end{array}\right] \xrightarrow[\substack{\downarrow \\ R_2}]{-R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -2 & 0 & -2 \\ 1 & 0 & 1 & 4 \end{array}\right]$$

$$\xrightarrow[\substack{\downarrow \\ R_3}]{-R_1+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -2 & 0 & -2 \\ 0 & -1 & 0 & -1 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & -1 \end{array}\right]$$

$$\xrightarrow[\substack{\downarrow \\ R_3}]{R_2+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[\substack{\downarrow \\ R_1}]{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Consistent

pivot column

Free variable

$x + z = 4$

$y = 1$

$x = 4 - z$

$y = 1$

$z =$ free to choose

$(4, 1, 0)$, $(3, 1, 1)$ $(4-z, 1, z)$

z

Infinitely many solns.

1.2

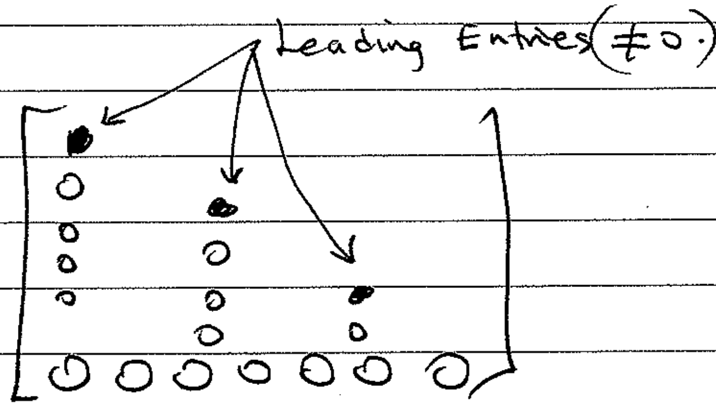
Leading entry of a row
= the leftmost non-zero entry

Echelon Form:

1. All zero rows must appear below all rows with a non-zero entry

2. Leading ~~word~~ entry of a row is to the left of the leading entry of a row below

3. For each leading entry of a row the entries below that are 0.



a) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ No Rule #2 is not satisfied

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ No 0's are not at the bottom

c) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 3 & 0 \end{bmatrix}$ No Rule #3 is not satisfied

d) $\begin{bmatrix} 3 & 5 & 2 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Echelon form.

RREF / RREM Reduced row echelon Form matrix

- Rules 1 – 3 above are satisfied (Echelon Form)

Further: 4) All leading entries are 1

5) Each column with a leading 1, has 0's in the rest of the column.

$$\left[\begin{array}{cccccc|c} 1 & 2 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$ is a RREF.

Ex 1

$$\begin{aligned} x + 3y - z &= 4 \\ 2x - y + z &= 6 \\ 3x + 2y &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 2 & -1 & 1 & 6 \\ 3 & 2 & 0 & 4 \end{array}\right] \xrightarrow[R_2 - 2R_1 \to R_2]{} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 0 & -7 & 3 & -2 \\ 3 & 2 & 0 & 4 \end{array}\right]$$

$$\xrightarrow[R_3 - 3R_1 \to R_3]{} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 0 & -7 & 3 & -2 \\ 0 & -7 & 3 & -8 \end{array}\right] \xrightarrow[R_3 - R_2 \to R_3]{} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 0 & -7 & 3 & -2 \\ 0 & 0 & 0 & -6 \end{array}\right]$$

Inconsistent so: STOP

First Do Echelon form; If consistent then RREF to save time.

Ex 2

$$\begin{aligned} x + 2y + 3z &= 0 \\ 2x - y - z &= 1 \\ 5x \quad\quad + z &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & -1 & -1 & 1 \\ 5 & 0 & 1 & 2 \end{array}\right] \xrightarrow[R_2 - 2R_1]{} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -5 & -7 & 1 \\ 5 & 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow[R_3 - 5R_1]{} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -5 & -7 & 1 \\ 0 & -10 & -14 & 2 \end{array}\right] \xrightarrow[R_3 - 2R_2 \to R_3]{} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -5 & -7 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Echelon form.

CAUTION: This SLE is consistent since there is No Row of inconsistency : $[0 \cdots 0 \mid c \neq 0]$.

A row $[0 \cdots 0 \mid 0]$ does NOT imply consistency, it is actually no information at all:

$$0 \cdot x + 0 \cdot y + 0 \cdot z = 0$$

NEXT TO RREF:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -5 & -7 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & \frac{7}{5} & -\frac{1}{5} \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R_1 - 2R_2 \to R_1]{} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{5} & \frac{2}{5} \\ 0 & 1 & \frac{7}{5} & -\frac{1}{5} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

PTO

$$\begin{array}{cc}
 & \begin{array}{cccc}\text{pivot} & \text{pivot} & \text{Free variable } z & \end{array}\\
\begin{array}{c}\text{Leading}\\ \text{Leading}\\ \\ \end{array} & \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{5} & \frac{2}{5} \\ 0 & 1 & \frac{7}{5} & -\frac{1}{5} \\ 0 & 0 & 0 & 0 \end{array}\right]
\end{array}$$

$$x + \frac{1}{5}z = \frac{2}{5}$$

$$y + \frac{7}{5}z = -\frac{1}{5}$$

Soln

$$x = \frac{2}{5} - \frac{1}{5}z$$

$$y = -\frac{1}{5} - \frac{7}{5}z$$

z = free variable.

So for every choice of z we take, we will get a solution.

- Consistent
- Infinitely many solutions

(Actually they all lie on a line)

we will see that.