Which equations are linear?

1.1
a. $xy = 1$ NO
b. $\sqrt{x} + \sqrt{y} = 7$ NO
c. $2x + 3y = 7$ YES
d. $e^x - e^y = 5$ NO
e. $x = 1$ YES
f. $x^2 - x + y = 4$ NO

Defn An equation of the form
$a_1x_1 + a_2x_2 + \dots + a_nx_n = b$ where $a_1, \dots a_n, b \in \mathbb{R}$
$x_1, x_2 \dots x_n$ variables
is called a linear Eq.

Defn A SLE (System of Linear Equations) is
a collection of one or more linear equations.

Ex
$$\left.\begin{aligned} 2x - y &= 4 \\ x + y &= -1. \end{aligned}\right\}$$ SLE in 2 unknowns x, y & 2 equations.

solve ↓

$x = 1$ and $y = -2$ $\quad (x, y) = (1, -2)$
one solution!

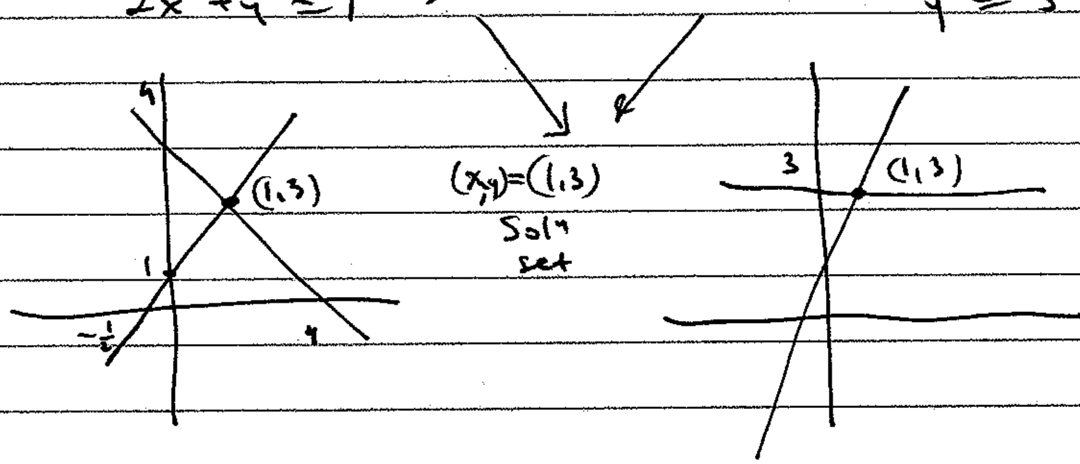
Defn A solution of a SLE in n-unknowns:
$x_1, x_2 \dots x_n$ is an ordered n-tuple of numbers
$$(x_1, x_2 \dots x_n) = (s_1, s_2 \dots s_n)$$
unknowns $\qquad$ real #'s or known quantities.

Defn Given an SLE, the set of all solutions is called the solution set.
(This is the set of all n-tuples of real #s satisfying all of the given equation(s).

Two SLE's are called equivalent if they have identical solution sets.

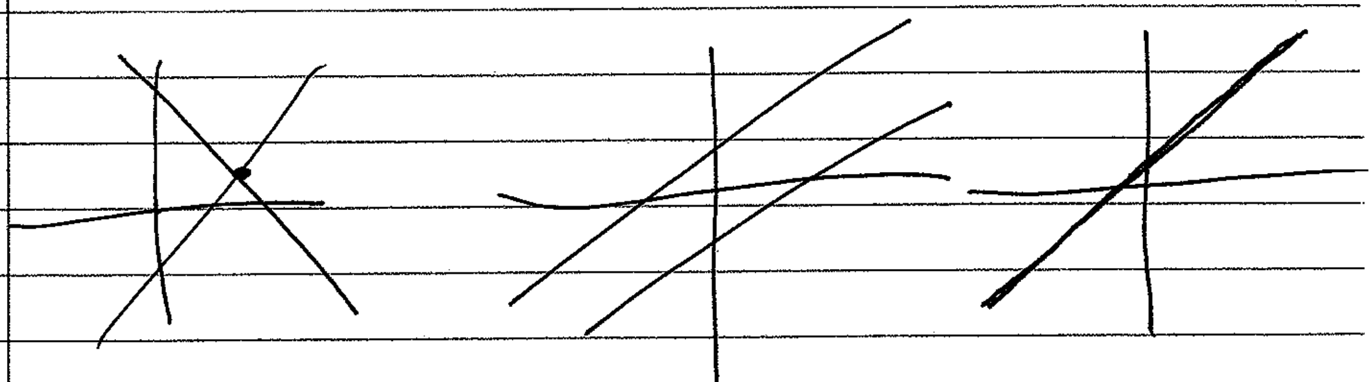
Ex

$$\left.\begin{aligned} x+y&=4 \\ -2x+y&=1 \end{aligned}\right\} \qquad \left.\begin{aligned} x-y&=-2 \\ y&=3 \end{aligned}\right\}$$



General
2 unknowns

$$\begin{aligned} ax+by&=A \\ cx+dy&=B \end{aligned} \qquad \underbrace{a,b,c,d}_{\text{not all zero}}, A, B \in \mathbb{R}.$$



THM Given an SLE, its solution set has

a) No solution OR ← Inconsistent
b) Exactly one solution OR ← Consistent
c) Infinitely many solutions.

Unique soln.

parametric sol'ns.

Ex

$0 \cdot x + 0 \cdot y = 1$ Inconsistent one equation

MATRICES of SLES

Given SLE

$$\left.\begin{aligned} 2x + 3y - z &= 4 \\ x - y + z &= 7 \\ x + y &= 8 \end{aligned}\right\} \circledast$$

Book doesn't use the bar.

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad \left[\begin{array}{ccc|c} 2 & 3 & -1 & 4 \\ 1 & -1 & 1 & 7 \\ 1 & 1 & 0 & 8 \end{array}\right]$$

Coeff matrix of $\circledast$ Augmented coeff. matrix of $\circledast$

Ex Solve the SLE

$$\begin{aligned} x + y - z &= -2. \\ x - y + z &= 0 \\ 2x + y + 3z &= 9. \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & -1 & 0 & 0 \\ 2 & 1 & 3 & 9 \end{array}\right] \xrightarrow[-R_1 + R_2 \to R_2]{} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -2 & 2 & 2 \\ 2 & 1 & 3 & 9 \end{array}\right]$$

Subtract eq #1 from eq #2

$$\xrightarrow[R_3 - 2R_1 \to R_3]{} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -2 & 2 & 2 \\ 0 & -1 & 5 & 13 \end{array}\right] \xrightarrow[-\frac{1}{2}R_2]{} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 5 & 13 \end{array}\right]$$

$$\xrightarrow[R_1 - R_2 \to R_1]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 5 & 13 \end{array}\right] \xrightarrow[R_3 + R_2 \to R_3]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 4 & 12 \end{array}\right]$$

$$\xrightarrow[\frac{1}{4}R_3]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_2 + R_3 \to R_2]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\boxed{\begin{aligned} x &= -1 \\ y &= 2 \\ z &= 3 \end{aligned}} \longleftarrow \left\{\begin{aligned} 1\cdot x + 0\cdot y + 0\cdot z &= -1 \\ 0\cdot x + 1\cdot y + 0\cdot z &= 2 \\ 0\cdot x + 0\cdot y + 1\cdot z &= 3 \end{aligned}\right.$$

The Solution, unique, consistent