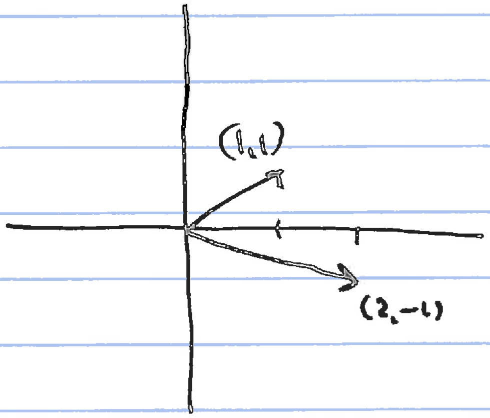
3/26/14

p1

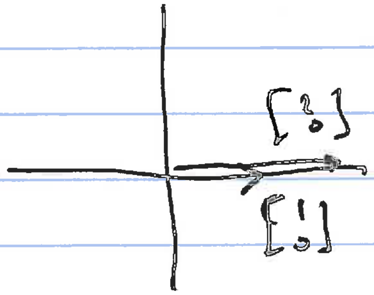
5.1

Ex

$$\begin{bmatrix}2&0\\0&-1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2x\\-y\end{bmatrix}.$$



$$\begin{bmatrix}2&0\\0&-1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}2\\-1\end{bmatrix}$$

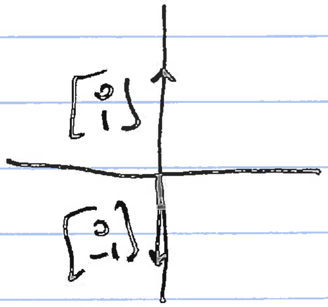


$$\begin{bmatrix}2&0\\0&-1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}2\\0\end{bmatrix}=2\begin{bmatrix}1\\0\end{bmatrix}$$

eigenvector

eigenvalue

$$\begin{bmatrix}2&0\\0&-1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}0\\-1\end{bmatrix}=-1\begin{bmatrix}0\\1\end{bmatrix}$$



Defn Let A be a given $n \times n$ matrix.
An eigenvector of A is a vector $\vec{v} \in \mathbb{R}^n$ s.t.

(i) $\vec{v} \neq 0$, and

(ii) There is a real number λ s.t.

$$A \cdot \vec{v} = \lambda \cdot \vec{v}$$

The number λ is called an eigenvalue of A. The vector v is called an eigenvector associated with λ.

Question ⓐ Is 5 an eigenvalue of $\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$?

Does there exist $\begin{bmatrix} x \\ y \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ s.t.

$$\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 5 \begin{bmatrix} x \\ y \end{bmatrix} \ ?$$

$$0x + 1y = 5x$$
$$3x + 2y = 5y$$

$$\begin{aligned} -5x + y &= 0 \\ 3x - 3y &= 0 \end{aligned} \implies \underset{\text{unique soln.}}{x = 0 = y}$$

$$\begin{vmatrix} -5 & 1 \\ 3 & -3 \end{vmatrix} \neq 0$$

5 is NOT an eigenvalue

ⓑ Is 3 an eigenvalue of $\begin{bmatrix}0&1\\3&2\end{bmatrix}$?

Does there exist $\begin{bmatrix}x\\y\end{bmatrix} \neq \begin{bmatrix}0\\0\end{bmatrix}$ s.t.

$$\begin{bmatrix}0&1\\3&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = 3\begin{bmatrix}x\\y\end{bmatrix} \ ?$$

$$\begin{cases} 0x + 1y = 3x \\ 3x + 2y = 3y \end{cases}$$

$$\begin{cases} -3x + 1y = 0 \\ 3x - y = 0 \end{cases}$$

$$\begin{bmatrix}-3 & 1\\ 3 & -1\end{bmatrix} \longrightarrow \begin{array}{cc} x & y \\ \left[\begin{matrix}1 & -\frac{1}{3}\\ 0 & 0\end{matrix}\right. & \left.\right] \\ & \uparrow \text{parameter} \end{array}$$

$$x - \tfrac{1}{3}y = 0$$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\frac{1}{3}y\\ y\end{bmatrix} = \begin{bmatrix}\frac{1}{3}\\ 1\end{bmatrix}\cdot y$$

$$\underset{y=1}{\begin{bmatrix}\frac{1}{3}\\ 1\end{bmatrix}}, \ \underset{y=3}{\begin{bmatrix}1\\ 3\end{bmatrix}},$$

$\overbrace{\text{eigenvectors.}}^{\text{associated to } 3=\lambda.}$, actually infinitely many but all parallel to each other.

Check: $\begin{bmatrix}0&1\\3&2\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}3\\9\end{bmatrix} = 3\begin{bmatrix}1\\3\end{bmatrix}.$ 3 is an eigenvalue.