3/12/14 (p1)

3.1 p 168 Exc. 12

$$\begin{vmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{vmatrix}$$

$$\begin{matrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \end{matrix}$$

$$= +4 \begin{vmatrix} -1 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{vmatrix} - 0 \begin{vmatrix} & \end{vmatrix} + 0 \begin{vmatrix} & \end{vmatrix} - 0 \begin{vmatrix} & \end{vmatrix}$$

$$= 4 \left(-1 \begin{vmatrix} 3 & 0 \\ 4 & -3 \end{vmatrix} + 0 \cdots + 0 \right)$$

$$= 4(-1)(3)\,|-3| = 4(-1)(3)(-3) = +36$$

determinant

not absolute value

Rule

$$\det \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ & a_{22} & 0 & 0 & 0 & 0 \\ & & a_{33} & 0 & 0 & 0 \\ & * & & \ddots & & 0 \\ & & & & & a_{nn} \end{bmatrix} = a_{11} \cdot a_{22} \cdot a_{33} \cdots a_{nn}$$

3.1 Exc # 24

$$\begin{vmatrix} a & b & c \\ 3 & 2 & 2 \\ 6 & 5 & 6 \end{vmatrix} = - \begin{vmatrix} 3 & 2 & 2 \\ a & b & c \\ 6 & 5 & 6 \end{vmatrix}$$

why?

$$\begin{vmatrix} a & b & c \\ 3 & 2 & 2 \\ 6 & 5 & 6 \end{vmatrix} = a\begin{vmatrix} 2 & 2 \\ 5 & 6 \end{vmatrix} - b\begin{vmatrix} 3 & 2 \\ 6 & 6 \end{vmatrix} + c\begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}$$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & \end{bmatrix}$$

$$= 2a - 6b + 3c$$

$$\begin{vmatrix} 3 & 2 & 2 \\ a & b & c \\ 6 & 5 & 6 \end{vmatrix} = -a\begin{vmatrix} 2 & 2 \\ 5 & 6 \end{vmatrix} + b\begin{vmatrix} 3 & 2 \\ 6 & 6 \end{vmatrix} - c\begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}$$

$$= -2a + 6b - 3c$$

3.2 Properties of Determinants

Effects of Row operations on det's.

THM Let A be an $n \times n$ matrix.

- If two rows of A are interchanged to obtain a matrix B, then $\det B = -\det A$.
- If one row of A is multiplied with k to obtain a matrix B then $\det B = k \det A$.
- If a multiple of one row of A is added to another row of A to obtain B then $\det A = \det B$.

Ex (a) $\begin{vmatrix} \pi & e & e^6 \\ 0 & 0 & 0 \\ -1 & \ln 7 & 4 \end{vmatrix} = 0$

(b)
$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 6 & 3 \\ 6 & 6 & 3 \end{vmatrix} = a$$

interchange row 2 & row 3

$$\|$$

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 6 & 3 \\ 6 & 6 & 3 \end{vmatrix} = -a$$

Then

$a = -a$

$\Downarrow$

$a = 0$.

Obs If a square matrix A has 2 identical rows, then $\det A = 0$.
Similarly if a square matrix A has 2 identical columns then $\det A = 0$.

Ex
$$\begin{vmatrix} 3 & 6 & 9 \\ 1 & 1 & 3 \\ 4 & 7 & 6 \end{vmatrix} = 3\begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 3 \\ 4 & 7 & 6 \end{vmatrix} \underset{R_2 - R_1 \to R_2}{=} 3\begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 4 & 7 & 6 \end{vmatrix}$$

$$\underset{-4R_1+R_3 \to R_3}{=} 3\begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & -1 & -6 \end{vmatrix} \underset{-R_2+R_3 \to R_3}{=} 3\begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & -6 \end{vmatrix} = 3(1)(-1)(-6)$$

$$= 18$$

Exc #8 p175

(Ex)
$$\begin{vmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{vmatrix} \underset{-2R_1+R_3 \to R_3}{=} \begin{vmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & -1 & -2 & 5 \\ -3 & -7 & -5 & 2 \end{vmatrix} \underset{R_2+R_3 \to R_3}{=} \begin{vmatrix} & & & \\ & & & \\ 0 & 0 & 0 & 0 \\ & & & \end{vmatrix}$$

$$\|$$

$$0$$

p 175 #12 $\begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{vmatrix} \underset{R_3 - 2R_4 \to R_3}{=} \begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ -3 & 0 & -2 & 0 \\ 4 & 2 & 4 & 3 \end{vmatrix}$

$= 3 \begin{vmatrix} -1 & 2 & 3 \\ 3 & 4 & 3 \\ -3 & 0 & -2 \end{vmatrix} \underset{-2R_1 + R_2 \to R_2}{=} 3 \begin{vmatrix} -1 & 2 & 3 \\ 5 & 0 & -3 \\ -3 & 0 & -2 \end{vmatrix}$

$= 3 \cdot (-2) \begin{vmatrix} 5 & -3 \\ -3 & -2 \end{vmatrix} = 3(-2) \cdot (-10 - 9)$

$= -6 \cdot -19 =$

$= 114.$

THM Let A, B be ~~an~~ $n \times n$ matrices

a) • $\det A = \det A^T$.

b) • $\det AB = \det A \cdot \det B$

c) • A^{-1} exists $\iff \det A \neq 0$

d) • A^{-1} exists $\implies \det A^{-1} = \dfrac{1}{\det A}$.

$d \Leftarrow b$ Ok: $A \cdot A^{-1} = I_n$

$\det A \cdot \det A^{-1} = \det I_n = 1.$

Exc #40 $\det A = -1$ 4×4

$\det B = 2$

$\det AB = \det A \cdot \det B = (-1)\,2 = -2.$

$\det B^5 = \det B \cdot B \cdot B \cdot B \cdot B = (\det B)^5 = 2^5 = 32.$

$\det 2A = (2^4) \cdot -1 = -16$

$\det A^T A = (\det A)^2 = 1$

$\det B^{-1}AB = \det A = -1.$

$$\left| \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right| = ad - bc$$

$$\begin{vmatrix} 2a & 2b \\ c & d \end{vmatrix} = 2(ad-bc)$$

$$\begin{vmatrix} 2a & 2b \\ 2c & 2d \end{vmatrix} = 4(ad-bc)$$