2.9 #14 p. 158

Find basis for the subspace spanned by

$$\begin{bmatrix}1\\-1\\-2\\3\end{bmatrix}, \begin{bmatrix}2\\-3\\-1\\4\end{bmatrix}, \begin{bmatrix}0\\-1\\3\\-2\end{bmatrix}, \begin{bmatrix}-1\\4\\-7\\7\end{bmatrix}, \begin{bmatrix}3\\-7\\6\\-9\end{bmatrix}$$

Suffices to find a basis for the column space of

$$\begin{bmatrix}1 & 2 & 0 & -1 & 3\\ -1 & -3 & -1 & 4 & -7\\ -2 & -1 & 3 & -7 & 6\\ 3 & 4 & -2 & 7 & -9\end{bmatrix} \longrightarrow \begin{bmatrix}1 & 2 & 0 & -1 & 3\\ 0 & -1 & -1 & 3 & -4\\ 0 & 3 & 3 & -9 & 12\\ 0 & -2 & -2 & 10 & 0\end{bmatrix}$$

$$\longrightarrow \begin{bmatrix}1 & 2 & 0 & -1 & 3\\ 0 & 1 & 1 & -3 & 4\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 4 & 8\end{bmatrix}$$

pivot

Basis for column space $\left\{ \begin{bmatrix}1\\-1\\-2\\3\end{bmatrix}, \begin{bmatrix}2\\-3\\-1\\4\end{bmatrix}, \begin{bmatrix}-1\\4\\-7\\7\end{bmatrix} \right\}$

We can extend Thm 8 of 2.3

<u>Thm</u>: The following are equivalent for an $\underline{\underline{n \times n}}$ matrix A:

(a) A is invertible
(b) columns of A form a basis of $\mathbb{R}^n$
(c) $\text{col } A = \mathbb{R}^n$
(d) $\dim \text{col } A = n$
(e) $\text{Rank } A = n$.
(f) Null space $A = \{\vec{0}\}$
(g) $\dim \text{nul } A = 0$.
(h) The rows of A form a basis of $\mathbb{R}^n$

End of $\underline{\underline{2.9}}$

3.1 Determinants

Recall if $ad-bc \neq 0$, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$ad - bc = \det A = \underbrace{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}_{2\times 2 \text{ determinant}}$$

Higher determinants are done by reduction to smaller size determinants.

$$\underset{\text{det}}{n\times n} \longrightarrow \underset{\text{det}}{(n-1)\times(n-1)} \longrightarrow \cdots \longrightarrow \underset{\text{dets}}{3\times 3} \longrightarrow \underset{\text{dets.}}{2\times 2}$$

Given a matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{n1} & a_{n2} & & & a_{nn} \end{bmatrix}$$

a_{ij} = ith row
jth column entry

A_{ij} = obtained by removing ith row (all of it)
& removing jth column of A.
(all of it)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 8 \\ 1 & -1 & 0 \end{bmatrix} \qquad A_{23} = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$$

Defn

$$C_{ij} = (-1)^{i+j} \det A_{ij} \qquad \underline{\text{cofactors}}$$

Def Given a $\underset{n\times n}{A}$ matrix A, $n \geq 2$

we define

$$\det A = \sum_{j=1}^{n} a_{1j} C_{1j}$$

$$= a_{11} C_{11} + a_{12} C_{12} + \cdots + a_{1n} C_{1n}$$

$$= (+1) a_{11} \det A_{11} - a_{12} \det A_{12} + a_{13} \det A_{13} \pm \cdots$$

$$\cdots + (-1)^{1+j} a_{1j} \det A_{1j} + \cdots + (-1)^{1+n} a_{1n} \det A_{1n}$$

Ex

$$\begin{vmatrix} 2 & -1 & 0 \\ 1 & 1 & 2 \\ -2 & 0 & 6 \end{vmatrix}$$

$$C_{ij} = (-1)^{i+j} \det A_{ij}$$

$$\text{Det} = a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13}$$

$$= + a_{11} \det A_{11} - a_{12} \det A_{12} + a_{13} \det A_{13}$$

$$= +\underbrace{2}_{a_{11}} \underbrace{\begin{vmatrix} 1 & 2 \\ 0 & 6 \end{vmatrix}}_{\det A_{11}} - \underbrace{(-1)}_{a_{12}} \underbrace{\begin{vmatrix} 1 & 2 \\ -2 & 6 \end{vmatrix}}_{\det A_{12}} + \underbrace{0}_{a_{13}} \underbrace{\begin{vmatrix} 1 & 1 \\ -2 & 0 \end{vmatrix}}_{\det A_{13}}$$

$$= +2(6-0) + 1(6+4) + 0 \cdot *$$

$$= 12 + 10 = 22.$$

Thm: Det A can be calculated by using the cofactor expansion along any row or any column:

$$\det A = \underset{\text{varying } j}{\sum_{j=1}^{n}} \underbrace{a_{ij}\, C_{ij}}_{\substack{i \text{ fixed} \\ i = \text{row \#}}} = \underset{\text{varying } i}{\sum_{i=1}^{n}} \underbrace{a_{ij}\, C_{ij}}_{\substack{j \text{ fixed} \\ j = \text{column \#}}}$$

Ex.

$$\begin{vmatrix} 2 & 3 & -1 \\ 4 & 1 & 6 \\ 5 & -2 & 4 \end{vmatrix} \qquad \begin{matrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{matrix}$$

$$(-1)^{i+j}$$

$$= -3\begin{vmatrix} 4 & 6 \\ 5 & 4 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ 5 & 4 \end{vmatrix} - (-2)\begin{vmatrix} 2 & -1 \\ 4 & 6 \end{vmatrix}$$

$$= -3(16-30) + 1(8+5) + 2(12+4)$$

$$= -3(-14) + 13 + 2 \cdot 16$$

$$= 42 + 13 + 32 = 87.$$

p168 #10

$$\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix}$$

$$\begin{vmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \end{vmatrix}$$

$$-0\begin{vmatrix} -2 & 5 & 2 \\ -6 & -7 & 5 \\ 0 & 4 & 4 \end{vmatrix} + 0\begin{vmatrix} 1 & 5 & 2 \\ 2 & -7 & 5 \\ 5 & 4 & 4 \end{vmatrix} - 3\begin{vmatrix} 1 & -2 & 2 \\ 2 & -6 & 5 \\ 5 & 0 & 4 \end{vmatrix} +$$

continue

$$+\ 0\begin{vmatrix} 1 & -2 & 5 \\ 0 & 0 & 3 \\ 5 & 0 & 4 \end{vmatrix}$$

$$= -3\begin{vmatrix} 1 & -2 & 2 \\ 2 & -6 & 5 \\ 5 & 0 & 4 \end{vmatrix}$$

$$\begin{vmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \end{vmatrix}$$

$$= -3\left(+5\begin{vmatrix} -2 & 2 \\ -6 & 5 \end{vmatrix} - 0\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 2 & -6 \end{vmatrix} \right)$$

$$= -3\left(5(-10+12) - 0 + 4(-6+4) \right)$$

$$= -3\left(5 \cdot 2 + 4 \cdot (-2) \right) = -6.$$

#

3x3

Fast way
$$\left|\begin{array}{ccc|cc} 1 & -2 & 2 & 1 & -2 \\ 2 & -6 & 5 & 2 & -6 \\ 5 & 0 & 4 & 5 & 0 \end{array}\right.$$

$$(-24 - 50 + 0) - (-60 + 0 - 16)$$

$$= -74 - (-76) = -74 + 76 = 2.$$

MIDTERM 1

A, A- } 20 %

88

B, B± } 44 %

72

C, C± } 28 %

54

D, D± } 6 %

36

F } 2 %