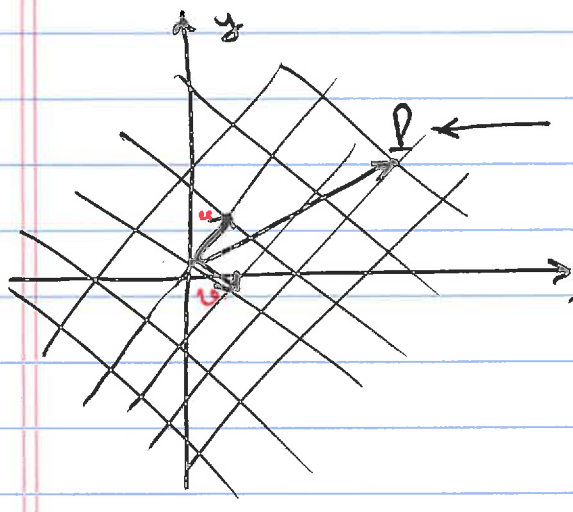
3/5/14 (p1)

(2.9)

Basis $\{\vec{v}_1, \dots \vec{v}_p\}$ in H a subspace of $\mathbb{R}^n$

if
- $v_1 \dots v_p$ span H, and
- $\{v_1, \dots, v_p\}$ is a linearly independent set

Coordinate systems



$3\vec{u} + 2\vec{v} = P$

coordinates of the point P according to the basis $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = u$, $\begin{bmatrix} 2 \\ -1 \end{bmatrix} = v$

$u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -1 \end{bmatrix} = v$

Defn: Given a basis $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \dots \vec{v}_p\}$ of a subspace H, and a point $q \in H$,

if one can write

$$q = c_1 \vec{v}_1 + c_2 \vec{v}_2 + \dots + c_p \vec{v}_p, \text{ then}$$

one defines the coordinates of q w.r.t. $\{\vec{v}_1, \vec{v}_2, \dots \vec{v}_p\}$ to be

$$[q]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_p \end{bmatrix}.$$

· Ex $P = 3u + 2v$

$$[P]_{B=\{\vec{u},\vec{v}\}} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Question How do we know for $q = c_1\vec{v}_1 + \cdots + c_p\vec{v}_p$ the numbers $c_1, c_2 \ldots c_p$ are uniquely defined? Suppose we can write q in two different ways with respect to $\{v_1, v_2 \ldots v_p\}$

$$q = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_p\vec{v}_p$$

$$q = d_1\vec{v}_1 + d_2\vec{v}_2 + \cdots + d_p\vec{v}_p$$

$$0 = q - q = (c_1 - d_1)\vec{v}_1 + (c_2 - d_2)\vec{v}_2 + \cdots + (c_p - d_p)\vec{v}_p$$

$\{v_1, \ldots, v_p\}$ linearly independent: $\begin{cases} c_1 - d_1 = 0 \\ c_2 - d_2 = 0 \\ \vdots \\ c_p - d_p = 0 \end{cases}$ #

Ex #6 p157 $b_1 = \begin{bmatrix} -3 \\ 2 \\ -4 \end{bmatrix}$ $b_2 = \begin{bmatrix} 7 \\ -3 \\ 5 \end{bmatrix}$

$$\vec{x} = \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}.$$

$$[\vec{x}]_{\{\vec{b}_1, \vec{b}_2\}} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$ where

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$$\begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix} = c_1 \begin{bmatrix} -3 \\ 2 \\ -4 \end{bmatrix} + c_2 \begin{bmatrix} 7 \\ -3 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -3 & 7 & 5 \\ 2 & -3 & 0 \\ -4 & 5 & -2 \end{array}\right] \xrightarrow[\substack{+ \\ R_1}]{R_1 + R_2} \left[\begin{array}{cc|c} -1 & 4 & 5 \\ 2 & -3 & 0 \\ -4 & 5 & -2 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|c} 1 & -4 & -5 \\ 2 & -3 & 0 \\ -4 & 5 & -2 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & -4 & -5 \\ 0 & 5 & 10 \\ 0 & -11 & -22 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|c} 1 & -4 & -5 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right].$$

$$\begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix} = 3 \begin{bmatrix} -3 \\ 2 \\ -4 \end{bmatrix} + 2 \begin{bmatrix} 7 \\ -3 \\ 5 \end{bmatrix} \checkmark$$

$$\left[\begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix} \right]_{\mathcal{B} = \{b_1, b_2\}} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

Prop: Given a linear subspace H, with two bases

$$B_1 = \{ \vec{b}_1, \vec{b}_2, .. \vec{b}_q \} \text{ and}$$
$$B_2 = \{ \vec{a}_1, \vec{a}_2 .. \vec{a}_p \}$$

for H. Then $q = p$, that is: the two bases must have the same number of vectors.

Defn Given a linear subspace H, the dimension of H is defined to be the number of vectors in every basis of H, provided that $H \neq \{0\}$.

$$\dim \{\vec{0}\} = 0.$$

Ex

$$\dim \operatorname{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\} = 2.$$

$$\dim \operatorname{span} \underbrace{\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}}_{\text{not basis}} = 2$$

spans $\mathbb{R}^2$

not lin independent

Exc #10 p158

$$A = \begin{bmatrix} 1 & -2 & -1 & 5 & 4 \\ 2 & -1 & 1 & 5 & 6 \\ -2 & 0 & -2 & 1 & -6 \\ 3 & 1 & 4 & 1 & 5 \end{bmatrix}$$

$$\cdots \longrightarrow \begin{bmatrix} 1 & -2 & -1 & 2 & 0 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[2R_2+R_1]{} \begin{bmatrix} 1 & 0 & 1 & 2 & +6 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$-2R_3 + R_1 \to R_1$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 6 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \longrightarrow \cdots \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

pivot pivot

free. x_3

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \\ 0 \\ 0 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Basis for null space $= \left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

dim null space = 1

Basis for column space $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ -6 \\ 5 \end{bmatrix} \right.$

sum = 5

5 = # columns

dim col A = 4

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Defn Given an $m \times n$ matrix A.

The rank of A is the dimension of the column space of A.

The dimension of null space of A nul A is called nullity.

RANK THM For an $m \times n$ matrix A (n = # columns)

$$\text{rank } A + \underbrace{\dim \text{nul } A}_{\text{nullity}} = n = \# \text{ columns.}$$

Exc #20 p159

$$8 = \underset{5}{\text{rank } A} + \underbrace{\dim \text{nul}(A)}_{3}$$

Will do #14 on Monday 3/10/14.