2/26/14 (p1)

- Midterm I Monday March 3, in this room!
  2229 SC
- Practice test is online
- Review Sunday March 2, <u>110 MLH</u>
  4:00–5:30 pm
- Material 1: 1, 2, 3, 4, 5, 7
  2: 1, 2, 3
- HW's

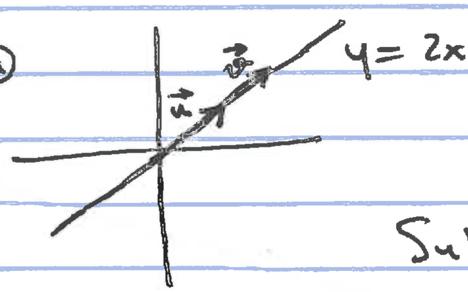
(2.8)

<u>Subspace</u>

<u>Defn</u> A set $H \subseteq \mathbb{R}^n$ is called a subspace of $\mathbb{R}^n$ if

- $\vec{u}, \vec{v} \in H \Rightarrow \vec{u} + \vec{v} \in H$.
- $\vec{u} \in H,\ c \in \mathbb{R} \Rightarrow c \cdot \vec{u} \in H$.

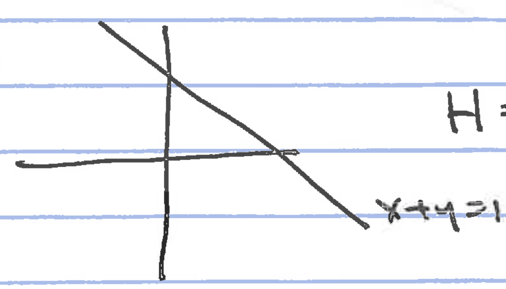
<u>Ex1</u> (a)



$\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \ y = 2x \right\} \subseteq \mathbb{R}^2$.

Subspace

(b)



$H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \ x + y = 1 \right\}$

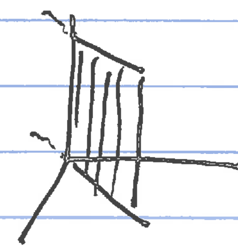
is not a subspace → (PTO)

- $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \in H \qquad \begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} \notin H.$

OR

- $0 \in \mathbb{R} \quad \begin{bmatrix}-2\\3\end{bmatrix} \in H \qquad 0 \cdot \begin{bmatrix}-2\\3\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \notin H.$

Ex $H = \left\{ \begin{bmatrix}x\\y\\z\end{bmatrix} \,\middle|\, x = y \right\}$



① Given: $\begin{bmatrix}x_1\\y_1\\z_1\end{bmatrix} \in H$ since $x_1 = y_1$ $\quad \begin{bmatrix}x_2\\y_2\\z_2\end{bmatrix} \in H$ since $x_2 = y_2$

then $\begin{bmatrix}x_1\\y_1\\z_1\end{bmatrix} + \begin{bmatrix}x_2\\y_2\\z_2\end{bmatrix} = \begin{bmatrix}x_1+x_2\\y_1+y_2\\z_1+z_2\end{bmatrix} \in H$ because $x_1 + x_2 = y_1 + y_2$

② $c \in \mathbb{R} \quad \vec{u} = \begin{bmatrix}x_1\\y_1\\z_1\end{bmatrix} \in H \Longrightarrow c\vec{u} = \begin{bmatrix}cx_1\\cy_1\\cz_1\end{bmatrix} \in H.$

$x_1 = y_1 \longrightarrow cx_1 = cy_1$

Ex 3 $\vec{w}_1, \vec{w}_2 \in \mathbb{R}^4$ given

$$H = \{ c_1\vec{w}_1 + c_2\vec{w}_2 \mid c_1, c_2 \in \mathbb{R} \}$$

H is a subspace of $\mathbb{R}^4$

Prop For any given set of vectors $\{\vec{w}_1, \vec{w}_2, \dots \vec{w}_p\}$ in $\mathbb{R}^n$, the span $\{\vec{w}_1, \dots \vec{w}_p\}$

= the set of all linear combinations of $w_1, \dots w_p$

is a subspace of $\mathbb{R}^n$.

Ex List All subspaces of $\mathbb{R}^3$:
- $\{\vec{0}\}$
- Every line thru $\vec{0}$
- Every plane thru $\vec{0}$
- all of $\mathbb{R}^3$

Ex Is $\begin{bmatrix} -1 \\ -4 \\ -8 \end{bmatrix}$ in the subspace generated by $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$?

Rewording: Does there exist c_1, c_2 s.t.

$$\begin{bmatrix} -1 \\ -4 \\ -8 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} ?$$

$$\left[\begin{array}{cc|c} 1 & 1 & -1 \\ 1 & 2 & -4 \\ 2 & 4 & -8 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & 1 & -3 \\ 0 & 2 & -6 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}\right]$$

- Conclude: $\begin{bmatrix} -1 \\ -4 \\ -8 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} - 3 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

Defn A subset B of a subspace H is called a basis if

- B is a linearly independent set, and
- B spans H

i.e.:

Every vector $\vec{u}$ in H is expressible as a linear combination of vectors in B.

Recall End of 1.7 Class notes:

If $H = \mathbb{R}^n$, $B = \{\vec{v}_1, \dots \vec{v}_p\}$, then

$$A = \left[\vec{v}_1 \ \vec{v}_2 \cdots \vec{v}_p\right] \xrightarrow{RR} RREF(A)$$

as columns

$RREF(A) = I_n$ then B is a basis.

$RREF(A) \neq I_n$ then B is not a basis.

2.8 Exc. #20 p151. 4 vectors in $\mathbb{R}^3$

$$A = \underbrace{\left[\quad\quad\right]}_{4\text{ columns}} \Big\}\, 3\text{ rows} \xrightarrow{RR} \underbrace{\left[\quad\quad\right]}_{3\times 4} \neq I_3, I_4$$

Not a basis

2.8 p151 #18

$$\begin{bmatrix} 1 & 3 & 5 \\ 1 & -1 & 1 \\ -3 & 2 & -4 \end{bmatrix}$$

Is it a basis? 2/26/14 P5

B

$\left\{ \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -4 \end{bmatrix} \right\} = B$

$$\begin{bmatrix} 1 & 3 & 5 \\ 1 & -1 & 1 \\ -3 & 2 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 5 \\ 0 & -4 & -4 \\ 0 & 11 & 11 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \not\sim I_3$$

B is Not basis

Not lin indep (not all columns have leading 1's)

Not spanning $\mathbb{R}^3$ (not all rows have leading 1's).

COLUMN SPACE

Given a $m \times n$ matrix A, the subspace of $\mathbb{R}^m$ spanned by the columns of A is called column space of A, denoted by col A.

NULL SPACE

Given a $m \times n$ matrix A, the subspace of $\mathbb{R}^n$

$$\{\vec{x} \mid A\vec{x} = 0\}$$ is called the

$\cap$ $\mathbb{R}^n$

Null space of A, denoted by nul A.

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p6

2.8 p151 #24 $A = \begin{bmatrix} 3 & -6 & 9 & 0 \\ 2 & -4 & 7 & 2 \\ 3 & -6 & 6 & -6 \end{bmatrix}$

- Find basis for the column space.

use 1.5 → • Find basis for the null space.

- Solve $A\vec{x} = 0$
- Find vector parametric soln

$$\begin{bmatrix} 3 & -6 & 9 & 0 \\ 2 & -4 & 7 & 2 \\ 3 & -6 & 6 & -6 \end{bmatrix} \xrightarrow[\text{HW}]{\text{RR.}} \begin{bmatrix} 1 & -2 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

pivot pivot

free x_2, x_4

$x_1 = 2x_2 + 6x_4$

$x_2 = \text{free } x_2$

$x_3 = -2x_4$

$x_4 = \text{free } x_4.$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 6 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$ vector parametric

A Basis of Null space $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}.$

Basis for column space:

Find pivot columns from RREF(A)

Then go to A & take the

corresponding columns.

method

Exc #24 p151 continue → RREF(A) has 1^{st}, 3^{rd} columns as pivot.

From A take 1^{st}, 3^{rd} column.

$$\text{Basis for col } A = \left\{ \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 9 \\ 7 \\ 6 \end{bmatrix} \right\}$$

Next page for the additonal example as promised in class.

ADDITIONAL EXAMPLE (As promised in class).
Find bases for col A and nul A where

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 4 & 6 \end{bmatrix}.$$

Sol^n RReduce $\begin{bmatrix} 2 & 3 & 2 \\ 1 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 6 \\ 2 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 6 \\ 0 & -5 & -10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 6 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \end{bmatrix}$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{pivots} & & \text{free} \end{array}$$

- $A\vec{x} = 0 \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} 2x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

Basis for null space $\left\{ \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right\}$

Basis for column space pivot columns of RREF(A) are #1, #2 columns.

Go to A #1, #2 columns

$\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ basis for col A.