MIDTERM · Next Monday 3/3 In class Here!

- 1.1, 2, 3, 4, 5, 7
  2.1, 2, 3
- Will post practice test by Wed.
- Review outside class
  Sunday 4pm – 5:30 will announce room.
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2.3

Ex $L : \mathbb{R}^3 \to \mathbb{R}^2$

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \underset{2\times 3}{\begin{bmatrix} 1 & -2 & 0 \\ 3 & 5 & 4 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} x-2y \\ 3x+5y+4z \end{bmatrix}}$$

Defn Given an $m \times n$ matrix A, one can define an associated map

$$L_A : \mathbb{R}^n \to \mathbb{R}^m$$

$$L_A\left(\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}\right) = \underbrace{\underset{m\times n}{A} \cdot \underset{n\times 1}{\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}}}_{m\times 1} \in \mathbb{R}^m .$$

7/24/14 p2

Obs

$$L_A(\vec{u}+\vec{v}) = L_A(\vec{u}) + L_A(\vec{v})$$
$$L_A(\underset{\text{real}}{c}\cdot\underset{\text{vector}}{\vec{u}}) = c\cdot L_A(\vec{u})$$
} Linear map.

L_A is a linear map; and vice versa any linear map from $\mathbb{R}^n$ to $\mathbb{R}^m$ is obtained by matrix multiplication.

Ex. $f(x) = 3x$ is a linear function

$[3][x] = [3x]$ ✓

CAUTION: $f(x) = 2x+1$ is not linear according to this defn.
(Differs from HS)

Affine map.

For any linear map $L(0)=0$, since

$$L(\vec{0}) = L(0\cdot\vec{u}) = 0\cdot L(\vec{u}) = \vec{0}.$$

2/24/14

(p3)

Let A be an $n \times n$ invertible matrix.

$$L_A(x) = A \cdot x$$

$$L_{A^{-1}}(x) = A^{-1} \cdot x$$

As functions, they are inverses of each other since

$$A^{-1} \cdot (A \cdot x) = (A^{-1} \cdot A) \cdot x = I \cdot x = x.$$

and

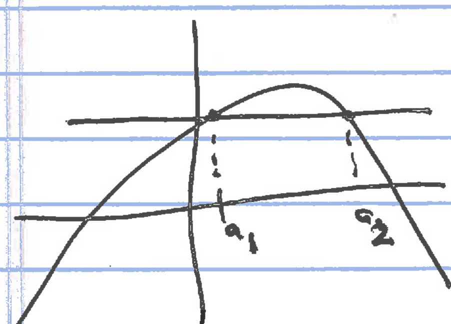
$$A \cdot (A^{-1} \cdot x) = (A \cdot A^{-1}) \cdot x = I \cdot x = x.$$

Precalculus: A function f is invertible if and only if it is one-to-one and onto.

Defn: A function $f: D \rightarrow R$ is called

- onto if for each $b \in R$, there is an $a \in D$ s.t. $f(a) = b$
- one-to-one: "if $f(a_1) = f(a_2)$ then $a_1 = a_2$, for all a_1 & a_2 in D."

Some Review of precalculus:



$a_1 \neq a_2$ $\quad f(a_1) = f(a_2)$

f is not 1-1.

Failing horizontal line test.

Ex $f: \mathbb{R} \to \mathbb{R}$

TYPO CORRECTION

$f(x) = x^2$ is not onto $\mathbb{R}$, since

$-1 \in \mathbb{R}$, but there is no x s.t.

$f(x) = x^2 = -1$.

Range $f = [0, \infty) \neq \mathbb{R}$
This is why this function is not onto $\mathbb{R}$.

Thm 8 p112 for $n \times n$ matrices

(*) Read and LEARN; review Chap I.

Thm 8 is not correct if non-square matrices are used:

(1) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$ columns are linearly indep but columns do not span $\mathbb{R}^3$

(2) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$ columns are not linearly indep columns span $\mathbb{R}^2$

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(3) $$\underbrace{\begin{bmatrix} 2 & 5 & 0 \\ 1 & 3 & 0 \end{bmatrix}}_{A} \begin{bmatrix} 3 & -5 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

not A^{-1}

$$\begin{bmatrix} 3 & -5 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 2 & 5 & 0 \\ 1 & 3 & 0 \end{bmatrix}}_{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 8 & 0 \end{bmatrix} \neq I_3$$

#4

$$\begin{bmatrix} -5 & 1 & 4 \\ 0 & 0 & 0 \\ 1 & 4 & 9 \end{bmatrix}$$ not invertible for sure, There is a row of zeros.

$$\begin{bmatrix} 1 & -3 & -6 \\ 0 & 4 & 3 \\ -3 & 6 & 0 \end{bmatrix}$$ Is it invertible? No easy/short answer.

see 2.2

$$\left[\underbrace{\begin{matrix} 1 & -3 & -6 \\ 0 & 4 & 3 \\ -3 & 6 & 0 \end{matrix}}_{A} \middle| \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I}\right] \longrightarrow \left[RREF(A) \,\middle|\, C \right]$$

No fast way, Do it in the long way

I_3 or $\neq I_3$ tells you whether A is invertible

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p115 #17 Can a square matrix with 2 identical columns be invertible?

Recall $(A^T)^{-1} = (A^{-1})^T$, if A^{-1} exists.

Let A be a matrix with 2 identical columns.

A^T is " " " " " rows.

RReduce $A^T \longrightarrow$ will get a row of zeros by subtracting one of the identical rows from the other

$RREF(A^T)$ has a row of zeros.

2/24/14 p6

A^T is not invertible

A is not "
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