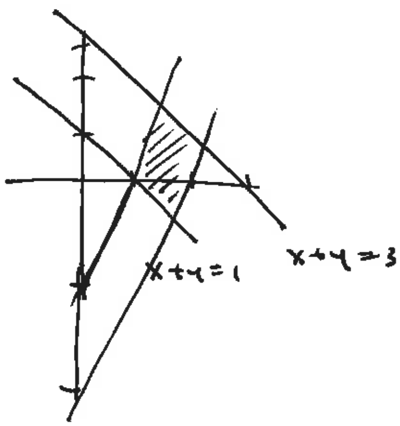
MATH 2850
Quiz 5
December 2, 2016

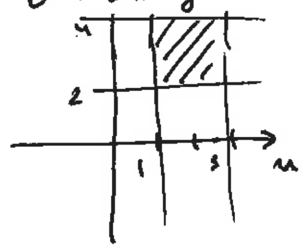
NAME. SOLUTION

1. Determine the value of $\iint\limits_D \frac{(x+y)^3}{2x-y}\,dA$ where D is the region enclosed by: $\begin{cases} x+y=1 \\ x+y=3 \\ 2x-y=2 \\ 2x-y=4 \end{cases}$ $\Rightarrow y = 2x-2$, $y = 2x-4$



$u = x+y$

$v = 2x-y$



$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3$$

$$du\,dv = 3\,dx\,dy$$

$$\iint\limits_D \frac{(x+y)^3}{2x-y}\,dx\,dy = \int_1^3 \int_2^4 \frac{u^3}{v}\,\frac{1}{3}\cdot dv\,du = \int_1^3 \left(u^3 \ln v \Big|_2^4\right) du \cdot \frac{1}{3}$$

$$= \int_1^3 \frac{1}{3}u^3 \cdot \underbrace{(\ln 4 - \ln 2)}_{\ln 2}\,du = \frac{u^4}{12}\ln 2 \Big|_1^3 = \frac{81-1}{12}\ln 2 = \frac{20}{3}\ln 2$$

2. Evaluate the following integral by using cylindrical coordinates. Sketch the domain of the integration in Cartesian coordinates.

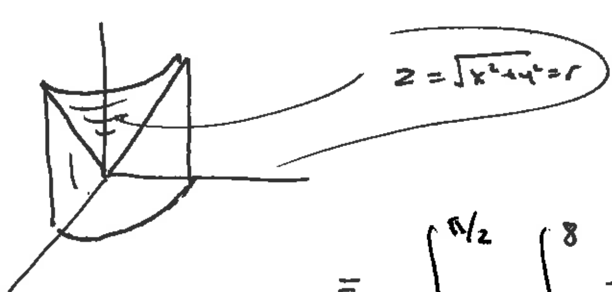
$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{x^2+y^2}} \cos[(x^2+y^2)^{3/2}]\,dz\,dy\,dx$$

$0 \le x \le 2$

$0 \le y \le \sqrt{4-x^2}$

$0 \le z \le \sqrt{x^2+y^2}$

$$= \int_0^{\pi/2} \int_0^2 \int_0^r (\cos r^3)\, r\,dz\,dr\,d\theta$$



$$= \int_0^{\pi/2} \int_0^2 z r \cos r^3 \Big|_{z=0}^{z=r} dr\,d\theta$$

$$= \int_0^{\pi/2} \int_0^2 r^2 \cos r^3\,dr\,d\theta$$

$$= \int_0^{\pi/2} \int_0^8 \frac{1}{3}\cos u\,du\,d\theta = \left(\frac{1}{3}\sin u \Big|_{u=0}^8\right) \int_0^{\pi/2} d\theta$$

$u = r^3 \quad du = 3r^2\,dr$

$$= \frac{\pi}{6}\sin 8$$