MATH 2850
Quiz 4
October 28, 2016

NAME. SOLUTION

1. Let $f(x,y) = y^2e^{3x} + xy^3$ be defined on $\mathbf{R}^2$.

$f(0,2) = 4 \cdot e^0 + 0 = 4$

a. Find the first order Taylor polynomial for f near $(a,b) = (0,2)$.

$f_x = 3y^2e^{3x} + y^3 \qquad f_x(0,2) = 12 + 8 = 20$

$f_y = 2ye^{3x} + 3xy^2 \qquad f_y(0,2) = 4$

$p_1(x,y) = 4 + 20(x-0) + 4(y-2) = 4 + 20x + 4y - 8$

$= -4 + 20x + 4y$

b. Use part (a) to approximate $f(0.01, 1.98)$.

$p_1(0.01, 1.98) = 4 + 20(0.01) + 4(1.98-2) = 4.12$

c. Find the second order Taylor polynomial for f near $(a,b) = (0,2)$.

$f_{xx} = 9y^2e^{3x} \qquad f_{xx}(0,2) = 36$

$f_{xy} = 6ye^{3x} + 3y^2 \qquad f_{xy}(0,2) = 24$

$f_{yy} = 2e^{3x} + 6xy \qquad f_{yy}(0,2) = 2$

$p_2(x,y) = 4 + 20x + 4(y-2) + \frac{1}{2}\left(36(x-0)^2 + 48(x-0)(y-2) + 2(y-2)^2\right)$

OR

$= 4 + \begin{bmatrix} 20 & 4 \end{bmatrix}\begin{bmatrix} x-0 \\ y-2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} x-0 & y-2 \end{bmatrix}\begin{bmatrix} 36 & 24 \\ 24 & 2 \end{bmatrix}\begin{bmatrix} x-0 \\ y-2 \end{bmatrix}$

2. Let $F(x,y) = x^3 + 2xy - x - y^2$. Find all critical points of F, and determine the nature of each (local max, min, saddle, or degenerate).

$\nabla F = (3x^2 + 2y - 1,\ 2x - 2y); \qquad \nabla F = 0 \iff 3x^2 + 2y - 1 = 0$

$2x - 2y = 0 \Rightarrow x = y$

$H_F = \begin{bmatrix} 6x & 2 \\ 2 & -2 \end{bmatrix}$

$3x^2 + 2x - 1 = 0$

$x = \frac{-2 \pm \sqrt{4+12}}{6}$

$= -1, \frac{1}{3}$

c.p:

$(-1,-1)$: $H_f(-1,-1) = \begin{bmatrix} -6 & 2 \\ 2 & -2 \end{bmatrix}$ $\left.\begin{matrix} \det = 8 \\ f_{xx} = -6 \end{matrix}\right\}$ local max at $(-1,-1)$

$(\frac{1}{3}, \frac{1}{3})$: $H_f(\frac{1}{3}, \frac{1}{3}) = \begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix}$ $\det = -8 < 0$ saddle at $(\frac{1}{3}, \frac{1}{3})$.