MATH 2850
Quiz 3
October 14, 2016

NAME. SOLUTION

1. a. Find the directional derivative of $f(x,y) = x^2 + 3xy + x\cos y$, at the point $\mathbf{a} = (3,0)$ in the direction parallel to the vector $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$.

$\nabla f = (2x + 3y + \cos y,\ 3x - x\sin y)$

$\nabla f(3,0) = (7, 9)$

$v = \frac{u}{\|u\|} = \frac{3i - 4j}{\sqrt{9+25}} = \frac{3}{5}i - \frac{4}{5}j$

$(D_v f)(3,0) = (7,9)\cdot\left(\frac{3}{5}, \frac{-4}{5}\right) = \frac{21-36}{5} = -3$

b. In which direction does f increase at the fastest rate at $\mathbf{a} = (3,0)$? What is the steepest rate of increase of f at $\mathbf{a} = (3,0)$?

f increases fastest in the direction $\frac{7i + 9j}{\sqrt{49+81}} = \frac{7i + 9j}{\sqrt{130}}$

Rate of steepest increase: $\sqrt{130}$

2. Consider the system:

$$\left.\begin{aligned} x^2u + xyu + uv + yv^2 &= 4 \\ y^2 + xy + uvxy &= 3 \end{aligned}\right\} \circledast$$

a. Explain why it is possible to solve x,y in terms of u,v (that is, there exists a solution $(x,y) = g(u,v)$) near the the point $(x,y,u,v) = (1,1,1,1)$.

$F(x,y,u,v) = (x^2u + xyu + uv + yv^2,\ y^2 + xy + uvxy)$

$$DF = \begin{bmatrix} 2xu + yu & xu + v^2 & x^2 + xy + v & u + 2vy \\ y + uvy & 2y + x + uvx & vxy & uxy \end{bmatrix}$$

$$DF(1,1,1,1) = \begin{bmatrix} \underbrace{\begin{matrix} 3 & 2 \\ 2 & 4 \end{matrix}}_{x,y} & \underbrace{\begin{matrix} 3 & 3 \\ 1 & 1 \end{matrix}}_{u,v} \end{bmatrix}$$

$\operatorname{Det} F_{x,y}(1,1,1,1) = \begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} = 8 \neq 0$

By Implicit F. Th. $\exists\ g(u,v) = (x,y)$ solving $\circledast$ locally.

b. Calculate Dg at $(x,y,u,v) = (1,1,1,1)$, and find $\frac{\partial y}{\partial u}$ at $(x,y,u,v) = (1,1,1,1)$.

$$Dg(1,1,1,1) = -\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}$$

$$= -\frac{1}{8}\begin{bmatrix} 10 & 10 \\ -3 & -3 \end{bmatrix} = \begin{bmatrix} -5/4 & -5/4 \\ 3/8 & 3/8 \end{bmatrix}$$

$\frac{\partial y}{\partial u}(1,1,1,1) = \frac{3}{8}$